# SCHOOL SCIENCE AND MATHEMATICS

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## SOME SUGGESTIONS TO MAP PUBLISHERS.

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A good map of the surface of the earth gives more knowledge of that surface in less space than any other known device and also presents the total of that knowledge more clearly than is possible by any verbal description. Nevertheless it remains an incontestable fact that few people are able to get clearly from maps the full and definite information which they contain. The latter statement is borne out by the fact that most people read all maps in the same way without any regard to the properties of the particular projection upon which the map before them is constructed. In general the reading public scarcely realizes that there are various projections upon which the world's surface may be mapped—each projection having certain advantages and also invariably being accompanied by certain disadvantages.

As the reading public is on the average just as intelligent as the smaller number who have had more or less training in the properties of map projections it would seem that something is wrong in our common school education, or in our methods of map printing, or in both. In general the mother tongue is so well taught that all who have had the advantages of the common schools are equally able to read any ordinary book—the appreciation of the contents varying of course with the previous experience and with the native ability of each individual. Everyone should be able to read maps with a facility equal to that with which he reads his native language.

This result it seems to the writer could be obtained by map publishers making some additions to the data on maps of large areas and by schools recognizing more clearly the few simple principles of map projections which should be thoroughly taught. The suggested changes in map printing are limited to maps of large areas because the errors in flat maps, while theoretically always present, are too minute to be noticed when small areas

are represented. The error grows rapidly with the extent of surface covered and in many projections becomes very great for a hemisphere or even a continent. The recent study of the relations of the Panama Canal to the rest of the world has led many people to extremely erroneous conclusions because they did not know the kind and amount of distortion incidental to the projection upon which their particular map of the world was based.

The simplicity of the changes to be suggested may be best appreciated by briefly noticing the very few ideas that need be taught in the public schools to enable anyone to fully understand the meaning and the limitations of common map projections. In fact the new style maps would be far more intelligible to the untrained man than those in use at present. He would really need nothing more than to lose his simple faith in the absolute accuracy of a flat map as a reproduction in miniature of the shapes and sizes of the countries of the earth's surface. But to give fullest appreciation of the properties of maps, the schools should teach thoroughly a few things that involve the general principles of map projections.

First will come in the primary school the fundamental conception of a map as a representation of the surface of the home locality and then of larger areas. Study of the surface of the earth as pictured on a globe should lead to the realization of the impossibility of the entirely correct representation of that surface on a flat map. Location of position on the globe by latitude and longitude should be followed by the presentation of two maps of the world with different schemes for drawing the parallels and the meridians and the statement that each map *records* true location by latitude and by longitude but *cannot* show correctly *all other* relations such as shapes, sizes, compass direction and shortest distance or true direction. Projections can now be presented on which some one thing is shown with entire accuracy at the expense of distortion of other things. An ideal series would be the Mercator projection for compass direction, the great circle map for shortest distance or true direction and an equal area projection for showing correctly the relative size of countries. Comparison with the globe in each case as well as with the other projections should be made to emphasize the general idea that accuracy in one detail means distortion in others. The course can now be completed by studying some of the general projections that try to minimize the distortion in everything

by sacrificing absolute accuracy in any one thing excepting the record of position by latitude and longitude, which all maps may give correctly. The names of only a few well-known projections need be taught.

Practically the brief summary above covers the essentials that are needed for the work of the elementary schools. The wealth of detail necessary and the number of years of school life through which this work should be distributed are minor problems for the schools and should not be confused with the main point of this paper, namely the suggestion of additional explanatory data to be printed on maps of large areas.

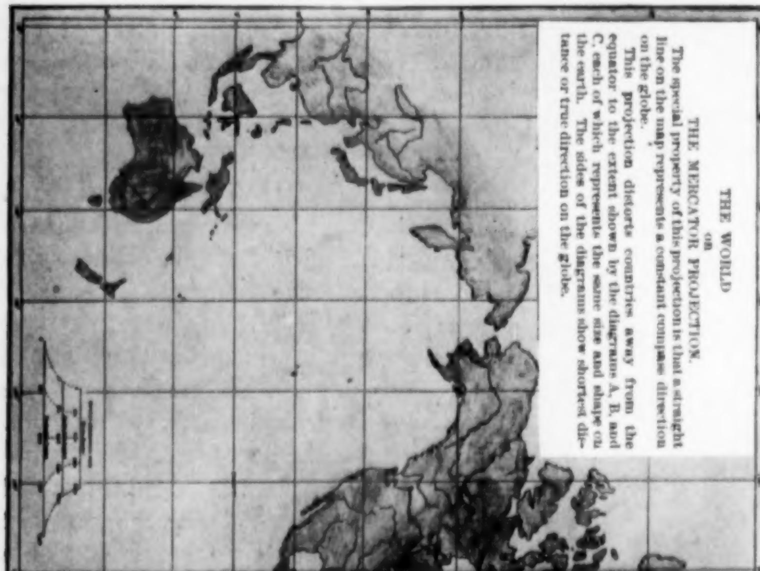
The additional data would serve to keep fresh in the minds of pupils the ideas of map projections obtained from the above course and should also be so definite as to be self-explanatory to any intelligent adult reader who uses the new maps.

The essential idea in the new data desired is that there should be index diagrams printed on every map showing the kind and amount of distortion incidental to the projection used. If a square of large size could be accurately laid off on the round earth it would be an ideal index figure, as distortion would be so easily noticed. The nearest approach to a square on the surface of a sphere is a regular quadrangle bounded by arcs of great circles. Such a figure bounded, we will say, by arcs of twenty degrees on each side would be identical in size and shape no matter upon what part of the earth's surface it were constructed, but reproduced in different parts of a map, it would vary in shape and size with the projection.

In figure 1 we have index diagrams (twenty degrees on a side) reproduced in different parts of a map of the world on the much-used Mercator projection. These widely differing figures, A, B, and C, represent areas of exactly the same size and shape on the earth.

We have been so accustomed to the gross exaggeration of the Mercator projection that I venture to assert that very few readers of this journal will see in the North America and Greenland of figure 1 the same distortion which strikes them so forcibly as they compare diagram C with diagram A and remember that each represents the same size and shape on the earth.

Pupils with a Mercator map of the world before them with index diagrams added as shown above would be interested in the method of construction of the projection. The following explanation is sufficient for common school purposes. The meridi-




ans which on the earth meet at the poles are drawn as parallel lines. As this makes them too far apart north or south of the equator the parallels of latitude are also drawn too far apart—the distance being constantly increased as the poles are approached so as to make at any point the distortion from N to S equal to the distortion from E to W. The result is that this

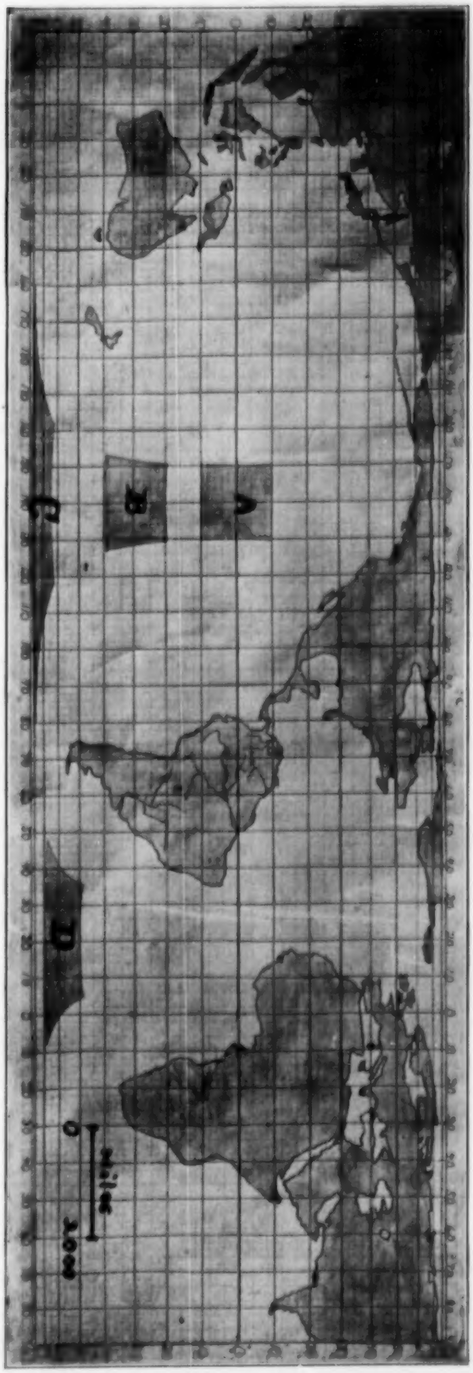



**Fig. 2. The World on the Cylindrical Equal Area Projection.**

Relative areas are correctly shown on this projection.
Distortion in shape is shown by the index diagrams which represent the same shape and size on the earth. They are of the same size on the map.
Scale of miles is true at the equator only.

projection shows compass direction by straight lines, but at the loss of a truthful representation of areas and shapes in high latitudes.

Fig. 1 is from a photograph of a map in which the distortion diagrams were worked out empirically by students as follows: A sheet of stiff paper was made to fit the surface of a large globe by cutting out narrow gores and then pasting on strips to hold the adjusted sectors in place. A regular quadrangle, each side twenty degrees on a great circle was next cut from the adjusted paper. This figure was then placed on various parts of the globe and the latitude and longitude of each corner and of three intermediate points on each side were noted in each case. These points were then laid off on the map and the sides of the index diagrams sketched in. The professional map maker would calculate his results but in the scale of the map as here reproduced the errors of the empirical method are not noticeable.

In class work I have found the cylindrical equal area projection (see Fig. 2) a good corrective (I had almost said antidote) for the errors of the Mercator projection. In this projection the meridians are drawn as equidistant parallel lines as in the Mercator, but the distance between parallels of latitude *decreases* toward the poles. The result is that relative areas are kept but shapes seem grotesque although they are in reality no more distorted than in the Mercator. The awkward shape of this map as a whole prevents its common use.

Lack of space prevents illustration of the distortion of the great circle projection and of others in more common use, but the suggestions for explanatory data on maps covering large areas would apply to any projections and to maps for popular use as well as to maps for schools.

In conclusion it should be noted that an appropriate legend is needed for maps with the index diagrams. The legend should create faith in maps, not destroy it, hence the distortion should be put in its true light as the necessary condition of attaining the special excellence of the particular projection. The legend should contain the following points:

(a) The name of the projection.

(b) A statement of the special property of the projection.

(c) An explanation of the index diagrams as showing necessary distortion to that end.

(d) A statement as to applicability of the scale of miles in

different portions of the map—or several scales given for particular parts of the map.

The Mercator map in Fig. 1 will serve as an illustration of the new map desired, not merely for index diagrams but also for general form of legend and for method of expressing the scale of miles in maps with much distortion.

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## A MOST EFFECTIVE METHOD OF DISCOURAGING GOOD TEACHING OF PHYSICS IN SECONDARY SCHOOLS.

By P. M. Dysart, Central High School, Pittsburg, Pa.

The following examination in physics was set by one of our leading Eastern colleges for women in the fall of 1910. The time limit of the examination was one and one half hours.

"1. Define the terms: acceleration, force, moment of a force, energy.

"2. Calculate the angular speed of the second, minute, and hour hands of a clock.

"3. State Archimedes' Principle. A piece of wood whose density is 0.6 floats on water. The volume of the wood is 40 cubic centimeters. What is the volume of the water displaced?

"4. Describe the method of propagation of sound.

"5. Describe the optical parts of a telescope. Draw a diagram showing the action of a telescope upon incident rays of light.

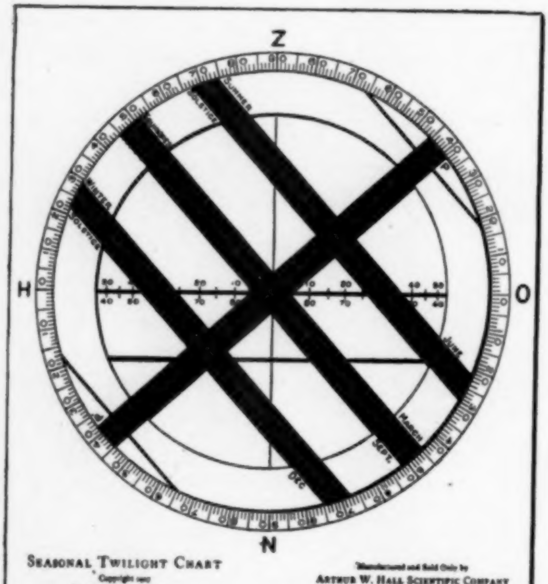
"6. Define the terms: ohm, volt, and ampere. What is the total electrical resistance of two wires of resistances $R_1$ and $R_2$ respectively when joined in parallel?"

The defects of this paper are so obvious that I shall not take time to point them out in detail. The most serious of them is that a pupil could make a creditable showing by learning by rote the definitions and formulæ of a text-book without comprehending a single one of them. From the character of the questions set, one would judge that the authorities of this college consider that demonstrations, laboratory work, real problems, and real teaching have no rightful place in a modern course in physics, that the (so-called) teacher's office is to hear the pupil repeat the *words* of the text-book. Moreover, even if no objection could be raised to the character of the questions, the number is certainly entirely too small to give a fair test of the pupil's mastery of physical principles. Unless the examiner be gifted with the power of omniscience. I fail to see how he could judge a pupil's fitness for college work by the character of his replies to two questions on mechanics, one on sound, one on light, and one on electricity. (I confess my inability correctly to classify the second question, and, therefore, do not include it.) It will be noticed that no question is asked concerning heat. This seems strange; for one would suppose that, on account of its bearing upon problems of heating, ventilation, and cooking, this subject would be of peculiar interest and importance to the girls of our schools.

## DAYTIME WORK IN ASTRONOMY.

By Sarah F. Whiting,
*Wellesley College.*

The Seasonal Twilight Chart is a device which developed from an exercise suggested by the question of a student. She said she had elected astronomy because she expected next summer to take the trip to the North Cape, and everyone would expect a college girl to be able to explain the midnight sun. To make this subject clear, students were set with compass and protractor to make projections of the definitive paths of the sun at different latitudes and seasons. It occurred to the demonstrator to have one revolving disk which could be set for any latitude. It can be seen from the cut that the Seasonal Twilight Chart




shows the sky in orthographic projection as it would look to an observer east, in the horizon plane. On the definitive paths of the sun are graduations for every twenty minutes. The twilight circle, eighteen degrees below the horizon, is shown in projection. With one of these in the hands of each member of the class many problems can be solved.

It is well at the beginning of this exercise to set up a full mounted globe, so that the circles looked at edgewise are projected in straight lines; also the Sun Path Model described some time ago in this journal. By this the instructor makes sure that the student conceives that he is viewing the celestial sphere, and not a flat surface.

With a full understanding of the theory of twilight, and of the meaning of the circles on the chart, the student may well set the chart for his own latitude, and find the times of sunrise and sunset, in summer and winter, the duration of twilight, the summer and winter elevation of the sun. Then let him set for the equator and the pole, for some well-known place within the tropics like Panama, for a place within the Arctic Circle like the North Cape, and answer question in reference to daylight and twilight and meridian altitudes of the sun, and the number of degrees north or south of the east and west points, of sunrise and sunset.

Finally, all from his own observations with this simple device he can summarize in reference to conditions from equator to pole, at equinoxes and solstices, and state at what latitudes twilights are longest and shortest.

For those who possess a McVicar Globe, an admirable mechanism for illustrating the same phenomena with a large revolving sphere, it is worth while to give a demonstration with this apparatus by way of review.

The telescopic observations of the moon, necessarily limited in number in a large class, can be advantageously supplemented by the use of photographs. If the school cannot possess the magnificent Atlas of the Moon published by the Paris Observatory, it can at least secure for each student sets of the moon pictures published by this Journal, and one or more of Colas Maps of the Moon (there is no other so good).

Measurements should be given by which a projection of the lunar hemisphere, which is turned toward the earth, with the meridians of longitude, can be drawn in the notebook. The lunar "seas" should first be sketched in from the map, then the photographs, each of which represents a telescopic observation, should be studied, compared with the map, the objects named from the key, and then put on the student's map in proper position.

The most interesting features only should be selected in order to avoid crowding the student's map; the two triplets of craters,

the pentagon of craters near the south pole, the mountain chains and craters near. If the photographs are at hand it is well to show each feature on the terminator both at waxing and waning moon. When the student gets his chance for telescopic vision, after such a study, he sees how perfectly photographs tell the story and yet how much more beautiful is the gleaming object itself. Moreover, he knows how quickly to identify what he sees.

It is not within the scope of this article to speak of out-of-door observations of the moon's motions which must be constantly carried on, and on which reports must be called for. But these observations become more intelligent if a plot of the moon's movements in the sky during a complete lunation is made from positions given in the American Ephemeris. Three sheets of centimeter paper are pasted together and to a scale of one degree to a millimeter, the position of the sun for every half month are first plotted in for the year. This gives on the graph the ecliptic. Then the moon's position for every day of some lunation is plotted and marked by a crescent which the student must be taught to carefully turn towards the sun, changing the position at the proper time. After making a plot the important thing is to inspect it, and discover its teachings.

With a millimeter scale (of degrees) in hand the student records answers to such questions as these: What is the moon's greatest latitude south and north? (this gives the inclination of its orbit); what is its daily motion?—this should be tabulated, its mean daily motion determined and the greatest and the least, (perigee and apogee); by how many days does the time recorded on the plot from new moon to new moon differ from the time of its return to the same hour circle? This shows the difference between the sidereal and synodical month, which should then be accounted for.

A similar plot of the motions of the planet Venus properly measured gives the phenomena of elongation, and of morning and evening star.

A plot of the path of Jupiter or Mars at the time of opposition with the principal stars near by upon it is an admirable preparation for outdoor observations of the retrogradation motions of these planets, and also shows the inclination of the orbits and the position of nodes.

The old time orrery was perhaps too complicated to be of great use, but in the laboratory a simple apparatus like the

Columbia Planetarium is an assistance to the student. This can be used by the demonstrator in the laboratory divisions and shows well the motions of an inferior planet with reference to the moving earth, also the motions of the earth with reference to a superior planet.

THE COLUMBIA PLANETARIUM.

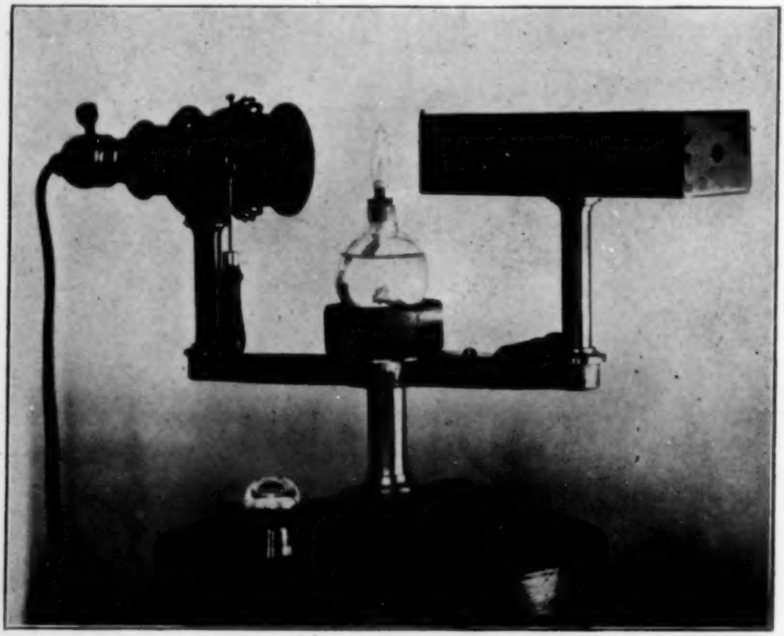
In this apparatus the student is not confused by too many objects. While the motions of an inferior planet are illustrated, the balls representing the morn and a superior planet can be removed.

No student can be favored with more than one or two observations of sun spots, but from drawings of the disk of the sun for a series of days he may easily arrive at great generalizations.

Let two sheets of graph paper be put together and Wolf's sun spot numbers in tabular form for a convenient period be given the student. Let them be platted with horizontal distances times and vertical distances sunspot numbers. When the plot is made let the times between maxima and also between minima be tabulated. The mean will give the sunspot period. Measurements of the times from minima to maxima and the reverse will reveal the law of increase and decrease of spots.

With sets of sunspot drawings for consecutive days like Sestini's and tracing linen, the student, placing the linen on the successive pictures of the sun's disk, can obtain a "composite" which will reveal the sunspot zones, the simple harmonic movement of spots across the solar disk, and the inclination of the solar axis.

The greatest modern achievements of astronomy have been won by the spectroscope, and none of these can be discussed with any degree of comprehension without some laboratory work in spectrum analysis. It would be well if in the high schools simple work with the spectroscope were substituted for some of the less "juicy" experiments now included in the beginning course in physics. As it is, no knowledge of the subject can be counted on even if a one year course in physics is a prerequisite to the course in astronomy.



**WILLSON'S SODIUM LINE REVERSER.**

**Nernst glower left, direct vision spectroscope right, wood alcohol flame saturated with salt behind the slit. As soon as the salted flame is inserted the dark sodium absorption line appears.**

Enough experiments should be performed to convince the student that every substance has its characteristic spectrum, as individual as the face of a human being. The sun spectrum with its absorption gaps should be mapped and the principal Fraunhofer lines made absolutely familiar, that they may be way marks for the placing of all lines discussed. To observe the lines of half a dozen of the lighter metals volatilized by the Bunsen flame, and the sun spectrum, and to map them to the same scale will make possible an elementary discussion of the great dis-

coveries of astro physics. If the reversal of the sodium line can be shown by the apparatus of Professor Robert Willson, and the spark spectra of hydrogen, helium, and cyanogen all the better.

Photographs of nebulæ and clusters in the northern and the southern sky can be studied, and sketched as a specimen is studied and drawn in the botanical laboratory. Sets of photographs of variable stars taken at two hour intervals studied by the students convince him that the light of the stars flickers up and down. A plot of the light curve of Nova Persei 1901, with the color of the star and the changing spectrum, put in at the proper times impresses the strange behavior of these objects in the sky.

There may be said to be two objects in teaching astronomy, to make astronomers and to train the intelligence. In elementary classes the latter is the chief object, and such laboratory exercises as those which have been outlined can be required with no peradventure of the weather and they certainly appeal to observation and require that individual thinking which is at the basis of mental development.

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## APPARATUS USED IN DAYTIME WORK IN ASTRONOMY.

12-inch celestial globes....................................................$8.00
 Robert Gair Company, Brooklyn, N. Y.
Colas' Maps of the Moon....................................................$2.50
 At present out of print. Plates owned by Poole Brothers, 116 Harrison St., Chicago.
Seasonal twilight chart....................................................$ .75
Solar calculator....................................................$15.00
 L. E. Knott Apparatus Co., Boston.
Gardner's Season Apparatus....................................................$36.00
 George S. Gardner & Co., 141 Clifton St., Rochester.
Columbia Planetarium....................................................$14.00
 Columbia School Supply Company, Indianapolis, Ind.
Kulliner Constellation Finder....................................................$5.00
 C. J. Kulliner, 505 University Place, Syracuse, N. Y.
The Heliodon.
 Central Scientific Company, Chicago.
Willson's Sodium Line Reverser.
Bunsen Burners and Simplex Spectroscopes.
 Wm. Gaertner & Co., Chicago, Ill.
The best small direct vision spectroscope is the miniature spectroscope of John Browning, $15.00, London, England. With this the Fraunhofer lines can be shown, also the coincidence of the sodium line and the D line, also the spectra of elements.
Lists of photographs of celestial objects are published by the Goodsell Observatory, Northfield, Minn., and the Yerkes Observatory, Williams Bay, Wis.

## A SECONDARY SCHOOL MATHEMATICS CLUB.

By Charles W. Newhall,
*Shattuck School, Faribault, Minn.*

The club whose workings are here described was organized in 1903 in Shattuck School, Faribault, Minn. So far as is known, this is the first attempt to conduct a club of this sort in a secondary school. That the experiment has been a success, the increasing enthusiasm for the work shown by the members of the club during the last eight years abundantly proves.

While Shattuck is a private school, which may make the matter of organization somewhat simpler, the course in mathematics is no higher than that in any good public high school, and the boys are the same age.

With some modifications, work could be done along similar lines, I believe, in any public or private high school. For that reason and because there have been several requests for information as to the workings of our club, our experience is here set down briefly for the benefit of secondary school teachers who may be interested.

The object of the club is to study certain interesting matters connected with mathematics which do not properly find a place in the usual classroom work. Such questions as the history of mathematics, its famous problems, unusual applications, mathematical puzzles, fallacies, and tricks are considered proper subjects for study, anything in fact that is capable of a mathematical solution or explanation and that promises to be entertaining. This is the main criterion, a subject must be interesting to find a place on our program; a club of boys will not thrive if the meetings are dull.

The club is made up of those members of the senior class who are taking the college preparatory course in mathematics, higher algebra, solid geometry and trigonometry, and numbers about fifteen. The instructor of the class is the leader, and chairman, *ex-officio,* and the only teacher usually present at the meetings. The club meets in their classroom on the evening of the weekly holiday. To make it easier for the boys to spare the time for these meetings it is understood that no preparation will be expected from them for the next day's regular lesson.

Each evening is devoted to some general subject, and reports are prepared on three or four special topics by as many members. This requires that each one of the members shall make

such a report about once in five or six weeks. At each meeting the program for the following meeting is announced, and thus those who are to take part have a week's time in which to prepare.

We have a reference library, freely accessible, in our recitation room, containing quite a number of books which will yield some information on the subjects in which we are interested. A list of the most useful of these books is given below. Besides the books, we have collected perhaps forty or fifty popular magazine articles on special topics in our field, as for example, one by Professor Newcomb on the "Fairyland of Geometry," others on "The Fourth Dimension," etc.

As boys of high school age are new to investigations of this kind it is necessary to help them in the preparation of their reports. They require rather explicit directions and suggestions as to the treatment of a given topic, what to include, and more especially what to leave out. They are given this help in the form of brief outlines of their topics together with references, often by pages, to two or three of the books available.

These reports are variously presented by the different members. Some of them talk extemporaneously from notes, making use of illustrations previously placed on the blackboard. Others prefer to mark certain pages or paragraphs, and to read directly from the books what they wish to tell us. Still others, on certain subjects, read carefully written papers, which they are not long in discovering may be submitted to other instructors in lieu of their monthly themes, or even elaborated into graduation theses.

The plan of our meetings is that each of these reports shall consume about fifteen minutes, and be followed by an informal discussion in which anyone may ask questions or raise objections to be met by the one presenting the report. If the leader can add anything of interest that has not been brought out he does so briefly, but the central idea of the whole scheme is to have the boys do most of the talking. The leader's function is to preside at the meetings, to keep the interest alive, and try to preserve that happy mien of informality which shall put the boys entirely at their ease, and yet maintain a certain dignity in the discussions.

Of course we can consider only such subjects as will be intelligible to secondary school pupils, and these only in an elementary and somewhat superficial way. To give an idea of the various subjects considered in this mathematics club, the program for last year is appended which may be regarded as a typ-

ical year's work. I may be pardoned perhaps if I describe the work of one or two of these meetings somewhat in detail in order to show the scope of our investigations.

On the evening devoted to the History of Geometry, for example, the principal topics are (1) The Beginnings of Geometry, (2) Early Greek Geometry, (3) The Golden Age of Greek Geometry, (4) Recent Developments in Geometry. In the first paper is considered the extent of geometrical knowledge among the early nations, the knowledge of astronomy among the Babylonians, and of mensuration, and surveying among the Egyptians, the Ahmes value of $\pi$, and his incorrect statements for the area of the trapezoid and isosceles triangle, the orientation of their temples, the "rope stretchers," and so on. In the consideration of the Early Greek Geometry the marked tendency of the Greek mind toward logic and the appreciation of form are referred to as accounting for this early development among them of geometry as a science. Thales is mentioned, and the theorems attributed to him, with their probable proofs. There is also a brief biography of Pythagoras, a description of his school, and the brotherhood of the Pythagoreans. In the third paper on the Golden Age of Greek Geometry, Euclid and his work are similarly discussed, his famous Elements described, and its influence on the modern study of geometry pointed out. The fourth paper considers recent contributions to Euclidean geometry and some of the elementary notions of the modern geometries.

When puzzles and fallacies are to be considered, the one presenting the report first proposes a fallacy or puzzle to the other members for solution; if they fail to solve the puzzle or point out the fallacy within a reasonable time, they are given a hint or the solution is shown them. Similarly in the case of Mechanical Puzzles, Card Tricks, Problems on a Chess Board, etc. In each case the conjurer after mystifying his audience, is required to give away his trick, and to explain the mathematical principle involved.

In introducing subjects like Our Number System or the Foundations of Geometry, it is necessary for the leader to do a larger part of the work, but even here it is possible for the students under direction to report on certain phases or details of the general topic. Again, in subjects like Non-Euclidean Geometry, Infinity, and the Fourth Dimension, we are on unfamiliar, and at the same time on rather dangerous ground, though it is still possible for the leader to point out the way, and with some care to

steer a safe course. Even if the students do not acquire perfectly clear notions about these rather vague subjects, it does no harm at least to set them thinking.

This is perhaps the most valuable result of the work of our club. It sets the boys to thinking and stimulates their imagination. They find that there is more of interest in the dry subject of mathematics than they had dreamed. Our investigations are not very profound to be sure, and will result in no contributions to the sum of mathematical knowledge, but the boys enjoy the meetings, and I think profit by them, and as these are the two distinct objects of this mathematics club, I feel that it has proved itself worth while.

As was said above, the treatment of the topics is suggested to the members of the club, by outlines which are prepared for them in advance. In order to show the extent of the help thus furnished, and as an example of how fully they are able to treat a subject, there is given below the outlines for two meetings.

THE HISTORY OF ARITHMETIC AND ALGEBRA.

*I. Among the Ancient Nations.*

- *a.* The Egyptians: Ahmes' papyrus, its antiquity, some of its problems, Egyptian fractions.
- *b.* Greek arithmetic and algebra: not so highly developed as geometry, Greeks interested in theory of numbers rather than calculation (examples). Computations on dust board (explanation). Diophantus and equations.
- *c.* Roman arithmetic: no algebra, arithmetic, only enough for commercial purposes. Use of abacus (illustrations).

*II. Among the Hindus and Arabs.*

- *a.* The Hindus (sixth and seventh century): Great computers, arithmetic and algebra their specialty as geometry for Greeks. Extent of their mathematical knowledge. Negative numbers, irrational numbers, zero. Their methods in arithmetic, false position, inversion. Examples of their poetically stated problems.
- *b.* The Arabs (eighth to the tenth century): brief history, great scholars of their time. Arabs as teachers and preservers of Greek and Roman culture, text-book writers. The name algebra, syncopated algebra.

*III. Early European Arithmetic.*

General education in the Middle Ages, schools of church

and monastery, universities (quadrivium, trivium), Rechenschulen of Hanseatic League.

Examples of four fundamental operations as performed in the Middle Ages; multiplication by Napier's rods, division by scratch method.

*IV. Development of Symbolic Algebra.*

Algebra originally rhetorical; work expressed fully in words, no symbols.

Syncopated algebra: abbreviations for the words, still few symbols, Diophantus (330 A.D.), the Arabs.

Symbolic Algebra: took modern form with Vieta (1600).

Examples of solution of an equation in:

1. Old rhetorical form.
2. Syncopated form.
3. As stated by Vieta.
4. In twentieth century symbols.

Advantage of our symbols.

*V. History of Common Symbols of Operation.*

Various devices used at different times to express powers, roots, equality, the four fundamental operations, etc., with their origin and dates.

First use of the present symbols.

## Number Systems and Numerals.

*I. Primitive Systems of Numeration.*

First notions of numbers, counting, etc., among animals, among savages.

Gesture symbols, finger reckoning. Early written symbols.

Systems in use among primitive races:

Decimal, Egyptian, Greek, Roman.

Quinary (scale of 5), certain aborigines of America.

Vicigesimal (scale of 20), South American tribes, etc.

Binary (scale of 2), New Zealand, Patagonian. Traces of other systems.

*II. Development of our Decimal Positional System.*

Due to ten fingers, how? Value of idea of position, of zero, discoveries of the Hindus (date). Advantages of our system (illustrated by multiplying two numbers in Roman notation).

Disadvantages of decimal as compared with duodecimal system.

*III. History of Arabic Numerals and Other Number Symbols.*
Babylonian, Egyptian, Greek symbols (illustrate). Awkwardness of Roman system.
History of development of so-called Arabic Numerals (Hindu origin).

*IV. Problems in Other Systems.*
Numbers written in other systems. Multiplication tables in other systems.
Problems in addition, division, etc., with numbers expressed in scale of 9, scale of 5, scale of 2, etc.
Problem in non-positional system like Greek or Roman, hence necessity for use of abacus.

*V. Number Systems of Algebra.*
Fractions: Egyptian unit fractions; Babylonian fractions, denominator always 60; Greek and Roman fractions; decimal fractions (recent date).
Irrational Numbers: first use, brief history, etc.
Negative Numbers: the same.
Imaginary Numbers: the same.

## APPENDIX A.

### THE MATHEMATICS CLUB OF SHATTUCK SCHOOL.

***Program for 1909-10.***

*FIRST MEETING: Algebra Fallacies.*
An informal consideration of certain proofs (?) that 2 equals 1, 1 equals 0, 1 equals —1, etc.

*SECOND MEETING: Our Number System.*
1. First notions of numbers.
2. Primitive numeration.
3. Development of decimal system.
4. The positional idea.

*THIRD MEETING: Number Systems and Symbols.*
1. History of our Arabic symbols.
2. Number symbols of other systems.
3. Nondecimal systems.
4. Some problems in a nondecimal system.
5. The duodecimal vs. the decimal.

*FOURTH MEETING: History of Arithmetic and Algebra.*
1. Among the ancient nations.
2. Among the Greeks and Romans.
3. Among the Hindus and Arabs.
4. In mediæval Europe.
5. The development of algebraic symbolism.

*FIFTH MEETING: Numerical Curiosities.*
1. Mystic properties of numbers.
2. Prime numbers, triangular numbers, squares, cubes, etc.
3. Magic squares.
4. Large numbers.
5. Number forms.

*SIXTH MEETING: Numerical Curiosities Continued.*
1. The number 9 and its properties.
2. Other curious numbers.
3. Mathematical short cuts.
4. Mental calculations.

*SEVENTH MEETING: Numerical Tricks and Puzzles.*
1. Numerical tricks.
2. Numerical puzzles and catch questions.
3. To discover a number thought of.

*EIGHTH MEETING: Geometrical Tricks and Puzzles and Mathematical Games.*
Informal consideration, no formal reports.

*NINTH MEETING: Card Tricks Involving Some Mathematical Principle of Number or Position.*
Problems on a chess board.

*TENTH MEETING: Foundations of Geometry.*
1. The assumptions.
2. Nature of space.
3. Definitions.
4. Logic of geometry.

*ELEVENTH MEETING: History of Geometry.*
1. Beginnings of geometry.
2. Early Greek geometry.
3. Euclid and his immortal elements.
4. Recent developments in geometry.

*TWELFTH MEETING: Famous Problems of Geometry.*
1. Squaring the circle.
2. The duplication of the cube.
3. Regular polygons and polyhedrons.
4. Famous problems of solid geometry.

*THIRTEENTH MEETING: Foundations of Geometry.*
1. Geometric assumptions.
2. The straight line, and how to draw one.
3. Non-Euclidean geometry.
4. Some criticisms of Wells geometry.

*FOURTEENTH MEETING: The Mathematics of Common Things.*
1. The mathematical principles of maps.
2. Optical illusions.
3. The carpenter's square.
4. Weighing and measuring.
5. Mathematical symmetry in nature.

*FIFTEENTH MEETING: The Fairyland of Geometry.*
1. The fourth dimension.
2. A visit to flatland.
3. A trip to infinity.
4. Curved space.

*SIXTEENTH MEETING: Higher Mathematics.*
1. History of trigonometry.
2. History of logarithms.
3. Calculus, and other pleasures to come.

## APPENDIX B.

Below are appended the titles of a few books and pamphlets selected from among the many available; they may be of interest

in connection with this account to teachers of secondary mathematics. Only publications in the English language are mentioned. Additional references will be found in Smith's Teaching of Elementary Mathematics, Young's Teaching of Mathematics, Withers' Parallel Postulate, White's Scrap Book of Elementary Mathematics, Ahren's *Unterhaltungen und Spiele.*

This list does not include text-books, nor articles in encyclopedias or magazines. Among magazines frequently containing such articles of interest are SCHOOL SCIENCE AND MATHEMATICS, *The Open Court, The Monist, Science, The Popular Science Monthly,* and *The Scientific American.*

### I. *Foundations and Criticisms.*

Common Sense of the Exact Sciences
    Clifford .................................................. Appleton
Grammar of Science
    Pearson .......................................... Black (London)
Foundations of Geometry
    Hilbert ..............................................Open Court
Foundations of Mathematics
    Carus ................................................Open Court
Space and Geometry
    Mach ..................................................Open Court
Euclid's Parallel Postulate
    Withers ..............................................Open Court
Foundations of Mathematics
    Russell .............................................. Macmillan
Euclid
    Frankland ........................................Wessel & Co.
Non-Euclidean Geometry for Teachers
    Halsted ................................................ Halsted
Non-Euclidean Geometry
    Manning ..........................................Ginn & Co.
Geometric Axioms (Popular Science Lectures, Second Series)
    Helmholtz ............................................. Appleton
Rational Geometry
    Halsted ..........................................Wiley & Son
The Thirteen Books of Euclid's Elements with Introduction and Commentary
    Heath ...............................Cambridge University Press
Mathematical Monographs
    Young ................................................ Longmans
Lectures on Elementary Mathematics
    Lagrange ............................................Open Court
Foundations of Mathematics
    Compto ..............................Harpers (out of print)
Theories of Parallelism
    Frankland ................................Cambridge Press

### II. *History.*

History of Mathematics
    Cajori ................................................ Macmillan

History of Elementary Mathematics
Cajori .................................................. Macmillan
History of Mathematics in the United States
Cajori ....................Government Printing Office, Washington
Short History of Mathematics
Ball .................................................. Macmillan
Primer of the History of Mathematics
Ball .................................................. Macmillan
Brief History of Mathematics
Fink ..................................................Open Court
Greek Geometry from Thales to Euclid
Allman ..................................Dublin University Press
History of Greek Mathematics
Gow ..................................Cambridge University Press
Euclid
Smith .................................................. Scribner
The Story of Euclid
Frankland ..............................................Wessel & Co.
Famous Problems of Geometry
Klein ..................................................Ginn & Co.
Mathematical Monographs
..................................................Heath & Co.
History of Modern Mathematics
Smith ..................................................Ginn & Co.
History of Teaching of Elementary Geometry
Stamper....................Teachers College, Columbia University
History of the Inductive Sciences
Whewell .................................................. Appleton
Portraits of Mathematicians
..................................................Open Court
The Teaching of Elementary Mathematics
Smith .................................................. Macmillan
Rara Arithmetic
Smith ..................................................Ginn & Co.
Sixteenth Century Arithmetic
Jackson ....................Teachers' College, Columbia University
The Hindu Arabic Numerals
Smith ..................................................Ginn & Co.
The Teaching of Geometry
Smith ..................................................Ginn & Co.

III. *Recreations.*

Scrap Book of Elementary Mathematics
White ..................................................Open Court
Mathematical Recreations
Ball .................................................. Macmillan
Mathematical Essary
Schubert .................................................. Open Court
The Canterbury Puzzles
Dudeney ..................................................Dutton & Co.
Paradoxes of Nature and Science
Harpson ..................................................Dutton & Co.
The Number Concept
Conant .................................................. Macmillan
Philosophy of Arithmetic
Brooks ..................Normal Publishing Company, Philadelphia

Reacreations in Science and Mathematics
  Ozanam ..................................................
  (Translated by Hutton; out of print; only second-hand copies obtainable)
Scientific Romances
  Hinton ................................Swan, Sonnenschein & Co.
Fourth Dimension
  Hinton ................................Swan, Sonnenschein & Co.
Flatland
  Anon. ......................................Little, Brown & Co.
Geometric Exercises in Paper Folding
  Row ............................................ Open Court
How to Draw a Straight Line
  Kempe ............................................ Macmillan

## PROVISIONAL REPORT OF THE NATIONAL COMMITTEE OF FIFTEEN ON GEOMETRY SYLLABUS.[1]

### Problems Involving Loci.

**(a) Phraseology.** While the committee does not wish to prescribe the exact phraseology of any definition, it would recommend greater care in the formulation of the definitions underlying the subject of loci. It is suggested that any definition used should be substantially equivalent to the following:

The locus of a point (or the locus of points) satisfying given conditions is a configuration such that:
(1) All points lying on the configuration satisfy the conditions;
(2) All points satisfying the conditions lie on the configuration.

It would seem desirable to make all proofs on loci conform to this definition. It is of course understood that the teacher will lead the pupil up to such a definition through varied forms of concrete description, such as "path of a point in motion," etc.

**(b) Motion in geometry.** It seems well to give some consideration to the place of motion in a well-rounded course in elementary geometry, and to bear in mind that this course is all the geometry to be studied by the majority of high school pupils. It has recently been urged by prominent European mathematicians that motion should be given a more prominent place at this stage. We may well recall that the space concepts dealt with in our usual courses in geometry are almost entirely to be described as static. There is in theorems and problems on loci a dynamic element that is of importance. The pupil is pretty fa-

[1]NOTE. The Geometry report is completed in this issue. It has already been reprinted in pamphlet form and sent to two hundred selected teachers for the purpose of securing further suggestions and criticisms prior to its final presentation at the meeting of the National Educational Association at San Francisco in July. The Committee would invite suggestions and criticisms from any reader of SCHOOL SCIENCE AND MATHEMATICS who may be interested. Such communications may be sent to any member of the Committee or directly to the Chairman, H. E. Slaught, 5535 Monroe Ave., Chicago, Ill.

miliar with motion as a concrete experience, and it seems of first-class importance to idealize some such concrete experiences, until they possess the precision of geometry.

For example, in a given plane we may consider in a way well described as a static configuration the perpendicular bisector of a line-segment joining two points; but when we consider this line as generated by a point moving in the plane in such a way that it is always equidistant from the two given points, we add a dynamic element.

As to phraseology, the expression "locus of points" suggests a static configuration, while the expression "locus of a point" emphasizes the dynamic element, and is equivalent in thought to the "path of a point moving with certain prescribed conditions." Both of these phases should have a place in the treatment of loci problems, and thus both forms of expression should be used, the one or the other being more suggestive in different cases. It is even desirable to use different forms in describing a given case to make clear the idea and to cultivate facility in expression.

(c) **Concrete nature of loci.** Contrary to the usual conception, the locus idea is one that may very easily be made concrete and brought down to the comprehension of young pupils. For example, the opening of a book or of a door suggests a variety of loci. The same may be said of many concrete illustrations easily accessible to the pupil.

In this way, loci problems may and should be introduced at certain stages of the subject. For example, in Book I: The locus of a point equidistant from two fixed points, equidistant from two intersecting lines, or from two parallel lines, or at a given distance from a fixed line. In the book on circles, the locus of all points equidistant from a fixed point, the locus of the centers of circles of fixed radius and tangent to a given line, the locus of the centers of all circles tangent to two parallel lines or two intersecting lines, and the locus of the vertices of all triangles having a common base and equal vertex angles. In solid geometry, the locus of points equidistant from a given point, from two given points, from a given plane, from two intersecting planes, from two parallel planes, etc.

**(d) Loci in problems of construction.** Important features of the construction problems in geometry are dependent upon loci considerations which should be emphasized in this connection. For example:

(1) To find the locus in the plane of all points equidistant

from three given points, it is necessary to determine the intersection of two loci both of which are straight lines.

(2) To find the locus in the plane of all points equidistant from a fixed point and at a given distance from a given line, it is necessary to find the intersection of two loci one of which is a straight line and the other a circle.

The discussion of the various possibilities in connection with such problems is one of the most valuable exercises for the pupil. For example, as to whether there are one, two, or no points fulfilling the conditions in the second example above. While it may be possible to solve and discuss such problems without using the term locus at all, yet this leads to roundabout and awkward explanations while the language of loci is elegant and concise.

Moreover, facility in the use of this language is not only desirable from the standpoint of the high school pupil but is of the utmost importance for those who may continue the study of geometry in college.

(e) **To summarize,** the locus idea is deserving of a careful and systematic treatment for the following reasons:

(1) It introduces a dynamic element through the consideration of the idea of motion.

(2) It presents an elegant language for the statement of those propositions on which nearly all of our problems of construction are based.

(3) It aids greatly in the cultivation of space intuition and in emphasizing the important concept of functionality.

(f) **Additional illustrations appropriate for use:**

1. Find the locus of all points at a fixed distance from the sides of a triangle, always measuring from the nearest point of a side.
2. Find the locus of points such that the sum of the squares of the distances from two lines intersecting at right angles is 100.
3. Find the locus of the vertices of a regular polygon of a given number of sides that can be circumscribed about a given circle.
4. Find the locus of the midpoints of the sides of regular polygons of a given number of sides that can be inscribed in a given circle.
5. Find the locus of all points from which a given line-segment subtends a given angle.
6. Find the locus of a point the sum of the squares of whose distances from two given points is constant.
7. Find the locus of a point the difference of the squares of whose distances from two given points is constant.
8. Find the locus of all lines drawn through a given point, parallel to a given plane.
9. Find the locus of a point in space equidistant from three given points not in a straight line.

## Algebraic Methods in Geometry.

The committee feels that the use of algebraic forms of expression and solution in the geometry courses may well be extended, with advantage to both algebra and geometry, and that this may be done without in any way encroaching upon the field of analytic geometry, which belongs to a later stage of development.

**(a) The notation should be more algebraic.** While it is not feasible or desirable to lay down hard and fast rules to standardize the notation of geometry, an examination of current texts makes it evident that some notations in common use are unnecessarily awkward when compared with the notations used in elementary algebra. The notation of geometry is, in general, improved by much use of lower-case letters to represent numerical values, leaving capitals to represent points. This notation is here called algebraic because the student will recognize the relations of equality and inequality much more readily in the familiar notation of algebra than if these relations are presented in a notation not used in algebra.

**(b) Algebraic statement of propositions.** Many of the theorems of geometry may be stated to advantage in algebraic form, thus giving definiteness and perspicuity and especially emphasizing the notion of functionality. This mode of expression can be made of much value to the student if he is required to translate into English all the symbols involved.

The following are illustrations of the algebraic statement of propositions:

(1) In any triangle, $a = bh/2$, where $a$ is the area, $b$ is the base and $h$ is the altitude.

(2) In a right triangle, $c^2 = a^2 + b^2$, where $c$ is the hypotenuse; and $a$ and $b$ are the sides including the right angle.

(3) In any triangle $c^2 = a^2 + b^2 \pm 2ap$, where $a$, $b$, $c$ are sides of the triangle and $p$ is the projection of $b$ on $a$.

(4) For any secant and tangent drawn from a point to a circle, we have $t^2 = sx$, where $t$ is the length of the tangent, $s$ is the length of the secant and $x$ is the length of the external part.

It is not intended to convey the impression that the usual statement of propositions should be replaced by the algebraic statements but rather that the student should be required to translate the one form of statement into the other. The algebraic statements are often superior to the usual statements in point of brevity and conciseness. Moreover, the algebraic statement prepares for the idea of functionality which is too little understood by persons who are not trained in mathematics be-

yond the high school course. That is to say, some appreciation of the influence of changing one part of a configuration on other parts of the configuration can often be gained readily from the algebraic statement.

(c) **Geometrical construction of formulas.** Some propositions can be proved simply and elegantly by methods involving algebra. It is somewhat usual in text-books on geometry to give a proof of the geometrical statement of such an algebraic formula as $(a + b)^2 = a^2 + b^2 + 2ab$, where $a$ and $b$ are the numerical measures of the line segments, but to neglect the geometrical construction of the formula. The latter seems to be the point of greatest importance. It is not additional evidence of the validity of the theorem that is sought. That is established in algebra. What is of first-rate importance is to give a geometrical picture of the formula, thus showing a certain geometrical interpretation and to have the student put the result into geometrical phraseology when $a$ and $b$ are line-segments.

The construction of line-segments $a + b$, $a - b$, and of areas $ab$, $(a+b)^2$, $(a-b)^2$, where $a$ and $b$ are line-segments should come early in the course. Later, when the requisite theorems are being developed, the further elementary expressions

$$ka,\ \frac{a}{k},\ \frac{ab}{c},\ \sqrt{ab},\ \sqrt{a^2+b^2},\ \sqrt{a^2-b^2},\ a\sqrt{k},$$

where $a$, $b$, and $c$ are line-segments and $k$ is a positive integer, should be constructed.

This interdependence of algebra and geometry is a matter of no small importance both historically and for subsequent mathematical work. It should be brought out by suitable exercises that the use of algebra often enables one to establish relations from which a geometrical construction can be made readily or to show the nature of a difficulty involved.

For example, to inscribe a square in a semicircle:

If $x$ represents the side of the square and $r$ the radius of the circle, we have at once from a right triangle that $r^2 = x^2 + x^2/4$ and hence $x = \pm {}^2/_5\sqrt{5}\, r$, which can be constructed from exercises given above.

(d) **Geometric exercises for algebraic solution.** Some exercises for algebraic solution, such as are found in many recent texts, should find a place in any course in geometry. For example, the following is a suitable exercise after the proposition stating that $a = bh$, where A is the area, $b$ and $h$ are sides of a rectangle:

The area of a rectangle is 480 square inches. Each side of the rectangle is increased 1 inch and, by this change, the area is increased 45 square inches. Find the sides of the rectangle.

Similarly, after the proposition pertaining to secants and tangents to a circle, the following is suitable:

A secant line which passes through the center of a circle of radius 10 is intersected by a tangent of length 15. Find the length of the external part of the secant.

Such exercises do much to unify geometry and algebra, and may well replace some of the usual exercises.

Finally, after the theorem on the volume of a frustum of a pyramid, a problem like the following has value as an algebraic exercise, although it is in no sense a real applied problem.

A pier is built of solid concrete construction, in the form of a frustum of a pyramid with square bases. The altitude is twice an edge of the lower base and the area of the lower base is four times that of the upper base. Find the dimensions of each base if the pier contains 600 cubic feet of solid concrete.

## *SECTION E. SYLLABUS OF GEOMETRY.*

### PREFACE TO LISTS OF THEOREMS.

(1) *Lists not exhaustive.* The lists of theorems which follow are not to be taken as exhaustive, and it is distinctly understood that theorems may be added at the discretion of the teacher. For example, the theorem on the existence of regular polyhedra may find a place in certain courses. Some theorems are omitted only with the understanding that they may be inserted as exercises for the student; some such possible exercises are:

In any triangle, the product of any two sides is equal to the product of the segments of the third side formed by the bisector of the opposite angle, plus the square of the bisector.

Upon a given line-segment corresponding to a given side of a given polygon, to construct a polygon similar to the given polygon.

To divide a given straight line-segment in extreme and mean ratio.

To find the area of a triangle in terms of its sides.

To construct a square having a given ratio to a given square.

The surface of a sphere is equivalent to the area of four great circles.

(2) *Logical order.* Although there is some indication of a possible logical order in the lists, there is no intention of specifying any definite order. It would be impossible to carry out as a whole precisely the order stated below.

It should be noticed that some logical arrangements would necessitate the insertion of the theorems omitted in this list.

Such an insertion is entirely in the spirit of this report, as is also any conceivable change in the order, except where specified explicitly in the report.

(3) *Subsidiary theorems.* A number of theorems omitted in the lists below may well be given as ordinary statements in the course of the text as corollaries, or as remarks, without the emphasis which attaches to formal theorems. Among such general statements which should by all means be made at the proper points are the following:

**No triangle can have more than one right angle or more than one obtuse angle.**

**The third angle of a triangle can be found if two are known.**

**An equilateral triangle is equiangular.**

**The square on a side of a right triangle adjacent to the right angle is equal to the square on the hypotenuse minus the square on the other side.**

**Through three points not in a straight line not more than one plane can be passed.**

**The areas of two spheres are to each other as the squares of their radii; their volumes as the cubes of their radii; (like statements for other solids).**

The number of such statements is exceedingly large and all of them could not be given in any syllabus. A large majority are at the present time stated, if at all, in the course of the reading matter, or in exercises, and not as explicit theorems. It is understood, and indeed expected, that these statements, together with many which are omitted from the lists of theorems below, should be treated in this manner.

(4) *Informal proofs.* The theorems given below under the heading: "Theorems for informal proofs," should be stated at the proper points in the text and in theorem form, or as postulates. Their proofs, however, can well be omitted where this omission is suggested, or be made exceedingly informal by the insertion of a single phrase which will give the proper suggestion for the proof. Many other theorems which are equally obvious are not stated because they occur more naturally as corollaries or as exercises. (See the preceding paragraph.)

Regarding the method of proof in general, while the demonstrations should remain as logical as they are at present, it is suggested that the formalities of logic, as such, be frequently dispensed with to a very considerable extent and that the propositions be frequently stated and proved in language resembling that to be found in any other mathematical text-book. This is, indeed, the style of many classical treatises, such as Legendre's or Euclid's. It is certainly satisfactory and there is no reason

why the proof should not remain quite as logical when the older style is followed.

The symbolic form of demonstration which appears in many texts should be regarded simply as a shorthand expression of a complete proof in ordinary English phraseology. The latter should be given by the student in all cases. The ability to pass from the symbolic form to ordinary English, that is, to translate the shorthand into the language of everyday life, should be constantly tested by the teacher, for the same reason that the formulas of algebra derive their real meaning and power from the thought content which the student can attach to them.

(5) *Arrangement for emphasis.* The main list of theorems is divided into several heads, each group being introduced by a theorem of suitable importance upon which the rest of the theorems in that group depend more or less closely. This arrangement has been selected in order to emphasize the importance of a few major propositions, namely, those which carry a maximum of applications and from which the rest can be derived, thus serving as a nucleus for the whole of geometry.

This effort to gain emphasis has been carried out still further by printing the theorems in different grades of type so that those of fundamental importance and of basal character are printed in black face type; those of considerable importance which are secondary only to the preceding ones are printed in italics. A number of other theorems are printed in Roman type, while the least important are printed in small type. The latter (small type theorems) may be omitted without serious danger, or they may be used as corollaries or exercises instead of receiving the emphasis which attaches to a theorem; in fact, probably no injury would result from a similar treatment of many of the theorems stated in Roman type.

The distinction in emphasis is desirable not only for guidance in omitting theorems in courses which are necessarily abbreviated, but it is also of the highest importance in courses in which all of the theorems are given. An orderly classification of theorems in the student's mind, a notion of the dependence of the minor theorems on the more basal ones and an appreciation of their relative importance is of the utmost direct value to the student and furnishes him with the only possible means of permanently retaining geometrical knowledge in usable form. The direct value mentioned arises both from the power acquired and also from the essential grasp of the subject, which

is the purpose of education. It is the fundamental characteristic of the human mind from which there is no escape that any impression of a vast field must have exactly such distinctions in emphasis as are outlined here for geometry. These statements and this arrangement are intended to be of assistance to the teacher in guiding him as to the emphasis to be laid upon theorems during the course and especially at the completion of a given book or chapter.

(6) *Trigonometric ratios.* Attention is called to the paragraphs under XIII, 2-4 on the computation of two-place tables of *sines, cosines,* and *tangents* from actual measurements, provided the pressure of time due to examining bodies is not too great. This work can be done with about the same amount of effort that is expended by the student on the ordinary geometrical theorems of the same class. Its importance is due to the fact that such a small table will really present the fundamental ideas of trigonometry and will enable the student to solve right triangles in the trigonometric sense.

(7) *Abbreviations.* In a large number of instances theorems are stated in condensed or abbreviated form and the statement of a number of theorems is often combined into one. This is done only for the purpose of reducing the length of this report. It is to be understood that such abbreviated statements are made only for the teacher and should not be presented to the student in this form. In particular, it is probably preferable to use words instead of letters in statements for high school pupils of such theorems as those in II, 1-4.

The numbers which follow each of the theorems are references to a syllabus prepared by a committee of the Association of Mathematics Teachers of New England, 1906.

(8) *Omissions.* The following list shows the omissions from the New England Syllabus:

*Plane geometry omissions*: C2, G12 (2nd part), G20, J13, L2, N6(2nd part), N13, P1, P2(see note to XV, 1), P6, P12, T2, T4, T5.

*Solid geometry omissions*: E7, E8, E9b, F12, F13, F14, K2, K3 (see note to I, 2), M6, M7, M8, M9, Q3, Q4, Q8b, Q9 (see VI, note), R1, R2 (see note to VI, 13), R9, R10, R13, R14 (see note to VI, 22), S3 (see preface, 3), S7.

THEOREMS OF PLANE GEOMETRY.

## I. Theorems for Informal Proof.

(The following theorems may be stated as assumptions, or may be given such informal proof as the circumstances may demand.)

1. All straight angles are equal.[45] [*]
2. All right angles are equal. [*]
3. The sum of two adjacent angles whose exterior sides lie in the same straight line equals a straight angle. [J1.]
4. If the sum of two angles equals a straight angle their exterior sides form a straight line. [J2.]
5. Only one perpendicular can be erected from a given point in a given line. [G3.]
6. The length of a circle (circumference) lies between the lengths of perimeters of the inscribed and circumscribed convex polygons. [P13.]
(It is recommended that this statement be used as a definition to be inserted at context.)
7. The area of a circle lies between the areas of circumscribed and inscribed convex polygons. [P14.]
(It is recommended that this statement be used as a definition to be inserted at context.)
8. Two lines parallel to the same line are parallel to each other. [*]
9. Vertical angles are equal. [J3.]
(Very informal proof sufficient.)
10. Complements of equal angles are equal. [N2.]
11. Supplements of equal angles are equal. [N3.]
12. The bisectors of vertical angles lie in a straight line. [J4.]
13. Any side of a triangle is less than the sum of the other two and greater than their difference. [*]
14. A diameter bisects a circle. [A5.]
15. A straight line intersects a circle at most in two points. [G6.]

## II. Congruence of Triangles.

1. **Any two triangles[46] ABC and A′B′C′ are congruent if:**

| | | | | |
|---|---|---|---|---|
| (1) | $a = a'$ | $b = b'$ | $C = C'$ | [A1.] |
| (2) | $a = a'$ | $B = B'$ | $C = C'$ | [A2.] |
| (3) | $a = a'$ | $b = b$ | $c = c'$ | [A3.] |
| (4) | $a = a'$ | $c = c'$ | $C = C' = 90°$ | [A4.] |

(State these in detail and in English. See preface, article 7.)

2. A triangle is determined when the following are given: (1) a, b, C; (2) a, B, C; (3) a, b, c; (4) a, c, $C = 90°$. [*]
(Synonymous to 1.)

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[45]Reference numbers are to the New England Syllabus. Where an asterisk [*] replaces the reference number, the theorem is not contained in that syllabus.

[46]In this syllabus the angles of a triangle ABC are denoted by the capital letters A, B, and C; the sides are denoted by small letters *a*, *b*, and *c*, where *a* is the side opposite the angle A, etc.

3. Construction of triangles from given parts; measurement of unknown parts by ruler and protractor. Given: (1) $a$, $b$, C; (2) $a$, B, C; (3) $a$, $b$, $c$; (4) $a$, $c$, C, possibly **two** solutions. [*]

(This is the fundamental, elementary idea of trigonometry.)

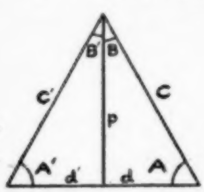
4. In any two triangles, if $a = a'$ and $b = b'$, either of the inequalities $c > c'$ or $C > C'$ is a consequence of the other. [O3, O4.]

### III. Congruent Right Triangles.

**1. Two right triangles are congruent if, aside from the right angles, any two parts, not both angles, in the one are equal to corresponding parts of the other.** [A4.]

(Very important: subcase of II, 1.)

2. If two oblique lines $c$ and $c'$ be drawn from a point in a perpendicular $p$ to a line AA′, cutting off distances $d$ and $d'$, then any one of the equalities, $c = c'$, $d = d'$, A = A′, B = B′, is a consequence of any other. [G5.]




3. *A diameter perpendicular to a chord bisects the chord, the subtended angle at the center, and the subtended arc; conversely, a diameter which bisects a chord is perpendicular to it.* [G5b, G8.]

(Corollary to 2. See also IV, 3.)

4. If two oblique lines $c$ and $c'$ be drawn from a point in a perpendicular $p$ to a line AA′, cutting off unequal distances $d$ and $d'$, then either of the inequalities $c > c'$, $d > d'$, is a consequence of the other. [O5, O6.]

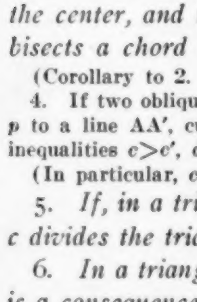
(In particular, $c$ is greater than $p$.)

5. *If, in a triangle ABC, $a = b$, the perpendicular from C on $c$ divides the triangle into two congruent triangles.* [*]

6. *In a triangle ABC, either of the equations $a = b$, $A = B$, is a consequence of the other.* [G1, G2.]

7. In a triangle ABC, either of the statements $a > b$, A > B, is a consequence of the other. [O1, O2.]

### IV. Subtended Arcs, Angles and Chords.




**1. In the same circle, or in equal circles, any one of the equations d = d′, k = k′, c = c′, O = O′, is a consequence of any other one of them.** [A6, 7, 8, 9, G9.]

2. Any one of the inequalities (see figure),

$$d < d', \quad O > O', \quad c > c', \quad k > k',$$

is a consequence of any other one of them. [O7, 8.]

3. *In any circle an angle at the center is measured by its intercepted arc.* [J8.]

(Only the commensurable case.)

4. If a circle is divided into equal arcs, the chords of these arcs form a regular polygon. [G12.]

5. To construct an angle equal to a given angle. [**J14.**]

(Regular polygons may be constructed approximately by means of a protractor. In the same way other approximate constructions may be introduced which depend upon the protractor.)

### V. Perpendicular Bisectors.

**1. The perpendicular bisector of a line-segment is the locus of points equidistant from the ends of the segment.** [**S1.**]

2. *To draw the perpendicular bisector of a given line-segment.* [**G14.**]

3. To erect a perpendicular at a given point in a line [*]

(Corollary to 2.)

4. To drop a perpendicular from a given point to a given line. [**D5.**]

(Corollary to 2.)

5. To bisect a given arc or angle. [**G15, 16.**]

(See III, 3.)

6. To inscribe a square in a circle. [**G18.**]

7. One and only one circle can be circumscribed about any triangle. [**G7.**]

(Construction to be given.)

8. Three points determine a circle. Two circles can intersect, at most, in two points; this will happen when the distance between their centers is less than the sum of the radii and greater than the difference of the radii. [**G7.**]

(Corollary to 7.)

9. Given an arc of a circle, to find its center. [*]

(Corollary to 7.)

10. A circle may be circumscribed about any regular polygon. [**G13, third part.**]

11. The perpendicular bisectors of the sides of a triangle meet in a point. [**T3.**]

(Corollary to 7.)

### VI. Bisectors of Angles.

**1. The bisector of any angle is the locus of points equidistant from the sides of the angle.** [**S2.**]

2. A circle may be inscribed in any triangle. [**G13, second part.**]

(Construction to be given.)

3. A circle may be inscribed in any regular polygon. [**G13, last part.**]

4. Of the inscribed and circumscribed regular polygons of $n$ and $2n$ sides for a given circle, to draw the remaining three polygons when one is given. [**G17.**]

5. The bisectors of the angles of any triangle meet in a point. [**T3.**]

(Corollary to 2.)

### VII. Parallels.

**1. When two lines are cut by a transversal the alternate interior angles are equal if, and only if, those two lines are parallel.** [**Half of D1,** 2.]

When two lines are cut by a transversal, the alternate interior angles are unequal, if and only if, the lines are not parallel.

(Synonymous to 1.)

2. When two lines are cut by a transversal the corresponding angles are equal, and the two interior angles on the same side of the transversal are supplementary if, and only if, the two lines are parallel. [**Half of D1, 2.**]

(Corollary to 1.)

3. Two lines perpendicular to the same line are parallel. [**D4, G4.**]

(Only one perpendicular can be let fall from a point without a line to that line. Synonymous to 3.)

4. A line perpendicular to one of two parallels is perpendicular to the other also. [**D3.**]

(Corollary to 1.)

5. *If two angles have their sides respectively parallel or respectively perpendicular to each other, they are either equal or supplementary.* [**J7.**]

6. Through a given point to draw a straight line parallel to a given straight line. [**D6.**]

7. *A parallelogram is divided into two congruent triangles by either diagonal.* [*]

8. In any parallelogram the opposite sides are equal, the opposite angles are equal, the diagonals bisect each other. [**D7.**]

(Corollary to 7.)

9. In any convex quadrilateral, if the opposite sides are equal, or if the opposite angles are equal, or if one pair of opposite sides are equal and parallel, or if the diagonals bisect each other, the figure is a parallelogram. [**D8.**]

### VIII. Angles of a Triangle.

**1. In any triangle the sum of the angles is two right angles.** [**J5(b).**]

2. In any triangle, any exterior angle is equal to the sum of the two opposite interior angles. [**J5(a).**]

(Corollary to 1.)

3. The sum of the interior angles of any polygon of $n$ sides is $2(n - 2)$ right angles. [**J6.**]

4. To inscribe a regular hexagon in a circle. [**G19.**]

To construct an angle of 60°. (Synonymous to 4.)

### IX. Inscribed Angles.

**1. An angle inscribed in a circle is measured by half of its intercepted arc.** [**J9.**]

2. *Angles inscribed in the same segment are equal to each other.* [*]

3. An angle inscribed in a semicircle is a right angle. [*]

4. The two arcs intercepted by parallel secants are equal. [**G11.**]

5. The angle between a tangent and a chord is measured by half the intercepted arc. [**J10.**]

6. The angle between any two lines is measured by half the sum, or half the difference, of the two arcs which they intercept on any circle, according as their point of intersection lies inside of, or outside of, the circle. [**J11, 12.**]

7. *The tangent to a circle at a given point is perpendicular to the radius at that point.* [**L1, 3.**]

8. For a given chord, to construct a segment of a circle in which a given angle can be inscribed. [**J15.**]

9. To draw a tangent to a given circle through a given point. [**L4.**]

10. The tangents to a circle from an external point are equal. [**G10.**]

(Corollary to 7).

### X. Segments Made by Parallels.

**1. If a series of parallel lines cut off equal segments on one transversal, they cut off equal segments on any other transversal.** [**D9.**]

2. *The segments cut off on two transversals by a series of parallels are proportional.* [**See N10.**]

(Only the commensurable case.)

3. *A line divides two sides of a triangle proportionally, the segments of the two sides being taken in the same order, if and only if it is parallel to the third side.* [**N1, 2.**]

(Only the commensurable case.)

4. To divide a line-segment into *n* equal parts or into parts proportional to any given segments. [**N9, 10.**]

5. To find a fourth proportional to three given line-segments. [**N11.**]

6. To find a mean proportional between two given line-segments. [**N12.**]

### XI. Similar Triangles.

**1. Two triangles ABC and A′B′C′ are similar if**

| | | | | |
|---|---|---|---|---|
| **(1)** | $A = A'$ | $B = B'$ | $C = C'$ | [**N3.**] |
| **or (2)** | $a = ka'$ | $b = kb'$ | $C = C'$ | [**N4.**] |
| **or (3)** | $a = ka'$ | $b = kb'$ | $c = kc'$ | [**N5.**] |

**where k is a constant factor of proportionality.**

(See preface, article 7.)

2. *Given a fixed point P and a circle C, the product of the two distances measured along any straight line through P, from*

*P to the points of intersection with C, is constant. This product is also equal to the square of the tangent from P to C if P is an external joint.* [**N18.**]

3. The bisector of any angle of a triangle divides the opposite side into segments proportional to the adjacent sides. [**Half of N6.**]

4. To construct a triangle similar to a given triangle. [*]

(Drawing triangles to scale; measurements of remaining parts to scale. Basal in trigonometry.)

### XII. Similar Figures.

*1. Polygons are similar if and only if they can be decomposed into triangles which are similar and similarly placed.* [**N7, 8.**]

2. Regular polygons of the same number of sides are similar. [**N14.**]

3. The perimeters of similar polygons are proportional to any two corresponding lines of the polygons. [**N15.**]

*4. The circumferences of any two circles are proportional to their diameters, thus, $c = 2\pi r$, where $\pi$ is constant.* [**P15.**]

($\pi = 3.14 \ldots.$ to be computed later.)

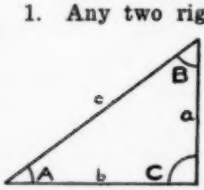
5. To construct a polygon similar to a given polygon. [*]

(Drawings to scale; maps, house plans; readings from drawings; plotting of measurements. Essential in surveying.)

### XIII. Similar Right Triangles.

(The committee feels that numbers 2, 3, 4 following should have a place where time for their discussion can be secured, which will doubtless be the case except under pressure from examining bodies.)

**1. Any two right triangles are similar if an acute angle of the one is equal to an acute angle of the other, or if any two sides of one are proportional to the corresponding sides of the other.** [*]




*2. For a given acute angle A, the sides of any right triangle ABC ($C = 90°$) form fixed ratios, called the sine ($a/c$), the cosine ($b/c$), the tangent ($a/b$).* [*]

3. Computation of a two-place table of sines, cosines, tangents from actual measurements. [*]

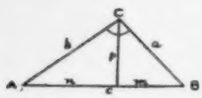
(Probably a two-place table for every 5° or 10°; to be done by students, preferably on squared paper.)

4. Solution of right triangles with given parts by use of the preceding table of ratios. [*]

(Height and distance exercises.)

### XIV. Right Triangles.

**1. In any right triangle ABC the perpendicular let fall from the right angle upon the hypotenuse divides the triangle into two similar right triangles, each similar to the original triangle.** [*]



2. The length of the perpendicular $p$ is the mean proportional between the segments $m$ and $n$ of the hypotenuse; i. e., $p^2 = mn$. [**P8.**]

3. Either side, $a$ or $b$, is the mean proportional between the whole hypotenuse $c$ and the adjacent segment $m$ or $n$; that is, $a^2 = cm$; $b^2 = cn$. [**P9.**]

**4. The sum of the squares of the two sides of a right triangle is equal to the square of the hypotenuse; $a^2 + b^2 = c^2$.** [**P10.**]

(It should be noticed that the proposition can be proved either algebraically or geometrically.)

5. In any triangle ABC, if B is less than 90°, then $b^2 = a^2 + c^2 - 2cm$; if B is greater than 90°, then $b^2 = a^2 + c^2 + 2cm$, where $m$ is the projection of $a$ on $c$. [**P17.**]

(See figure under 2.)

6. Given the radius of a circle and a perimeter of an inscribed regular polygon of $n$ sides, to find the perimeter of the regular circumscribed polygon of $n$ sides and the perimeter of the regular inscribed polygon of $2n$ sides. [*]

7. To calculate $\pi$ approximately. [*]

### XV. Areas.

**1. The area of a rectangle is the product of its base and its altitude; i. e., a = bh.** [**P1, 2, 3.**]

(This formula may be taken as the definition of area.)

2. *The area of a parallelogram is the product of its base and its altitude; i. e., $a = bh$.* [**P4.**]

3. *The area of a triangle is one half the product of its base and its altitude; i. e., $a = \frac{1}{2} bh$.* [**P5.**]

4. *Parallelograms or triangles of equal bases and altitudes are equivalent.* [**C1.**]

5. *The area of a trapezoid is one half the product of its altitude and the sum of its bases; i. e., $a = \frac{1}{2}(b_1 + b_2)h$.* [**P7.**]

6. The areas of similar triangles or polygons are proportional to the squares of corresponding lines. [**N16, 17.**]

7. The area of a regular polygon is one half the product of its perimeter and its apothem. [**P11.**]

8. *The area of any circle is one half the product of its circumference and its radius; i. e., $a = \pi r^2$.* [**P16.**]

9. The areas of two circles are proportional to the squares of their radii. [*]

(May be treated as suggested in preface, article 3.)

10. To construct a square equivalent to the sum of two given squares. [*]

(Pythagorean proposition.)

11. To construct a square equivalent to a given rectangle. [**C3.**]

(Mean proportional. See X, 6.)

THEOREMS OF SOLID GEOMETRY.

In this part the same general principles apply as were stated in the preface above.

Throughout, but particularly in divisions I and II below, very great emphasis should be laid upon the student's real grasp of the conceptions, of the space figures, and of the significance of the theorems. While the theorems in division I will be seen to need little or no suggestion of proof, it is a mistake to suppose that they can be hastened over; on the contrary, even in these, the teacher should spare no pains to make sure that the student's mental picture is quite vivid, resorting to formal proof when necessary. To this end, illustrations, figures, models, various forms of presentation, and all such aids are legitimate throughout the course in solid geometry.

**I. Theorems for Informal Proof.**

1. If two planes cut each other, their intersection is a straight line. [**S4.**]

2. Two dihedral angles have the same ratio as their plane angles. [**K2, 3, 4.**]

(Equivalent to K3.)

3. Every section of a cone made by a plane passing through its vertex is a triangle. [**M4.**]

4. Every section of a cylinder made by a plane passing through an element is a parallelogram. [**M2.**]

5. The area of a sphere lies between the areas of circumscribed and inscribed convex polyhedrons. [*]

(It is recommended that this statement be used as a definition to be inserted at context.)

6. The volume of a sphere lies between the volumes of circumscribed and inscribed convex polyhedrons. [*]

(It is recommended that this statement be used as a definition to be inserted at context.)

7. The projection of a straight line upon a plane is a straight line.

**II. Corollaries from Plane Geometry.**

(The ability to make the transfer from plane geometry to solid geometry, and vice versa, in forming conceptions and in logical deductions is of the utmost importance. The following

theorems are easily reducible to plane geometry in at most two or three planes. The intention is that careful proofs be given, but the student should see that these theorems result immediately from known theorems of plane geometry.)

1. The intersections of two parallel planes with any third plane are parallel. [**F1.**]

2. A plane containing one and only one of two parallel lines is parallel to the other. [**F7.**]

3. If a straight line is parallel to a plane, the intersection of the plane with any plane drawn through the line is parallel to the line. [*]

4. Through a given point only one plane can be passed parallel to two straight lines not in the same plane. [**F10.**]

**(Derived from 2.)**

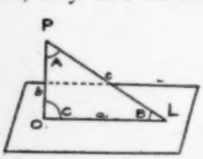
5. Through a given straight line only one plane can be passed parallel to any other given straight line in space, not parallel to the first. [**F11.**]

**(Derived from 2.)**

6. Through a given point only one plane can be drawn parallel to a given plane. [**F9.**]

**(Synonymous to 4.)**

7. If a perpendicular PO be let fall from a point P to a plane L, any one of the equalities




$$a = a', c = c', B = B', A = A'$$

is a consequence of any other of them; and any one of the inequalities

$$a > a', \ c > c', \ B < B', \ A > A'$$

is a consequence of any other one of them. [**O10, H7. See also S8.**]

8. The perpendicular PO is shorter than any oblique line. [**O9.**]

9. Two straight lines are parallel to each other if and only if they are both perpendicular to some one plane. [**F2, 3.**]

10. If two straight lines are parallel to a third, they are parallel to each other. [**F4.**]

**(Derived from 9.)**

11. Two planes are parallel to each other if and only if they are both perpendicular to some one straight line. [**F5, 6.**]

**(Derived from 9.)**

12. The locus of points equidistant from the extremities of a straight line is a plane perpendicular to that line at its middle point. [**S5.**]

13. If two straight lines are intersected by three parallel planes, their corresponding segments are proportional. [**See M1.**]
(**Draw the traditional auxiliary figure.**)

14. The locus of points equidistant from two intersecting planes is the figure formed by the bisecting planes of their dihedral angles. [**S6.**]

### III. Planes and Lines.

1. **If a straight line is perpendicular to each of two other straight lines at their point of intersection, it is perpendicular to every line in their plane through the foot of the perpendicular.** [**E1.**]

2. Every perpendicular that can be drawn to a straight line at a given point lies in a plane perpendicular to the line at the given point. [**E2.**]
(**Corollary to 1.**)

3. Through any point only one plane can be drawn perpendicular to a given line. [**E5.**]
(**Corollary to 1, and II, 11.**)

4. Through a given point only one perpendicular can be drawn to a given plane. [**E6.**]
(**Corollary to 1.**)

5. If two angles have their sides respectively parallel and lying in the same direction, they are equal, and their planes are parallel. [**K1, F8.**]

*6. If a line meets its projection on a plane, any line of the plane perpendicular to one of them at their intersection, is perpendicular to the other also.* [*]

7. **Between any two straight lines not in the same plane, one and only one common perpendicular can be drawn, and this common perpendicular is the shortest line that can be drawn between the two lines.** [**E12.**]

*8. Two planes are perpendicular to each other if and only if a line perpendicular to one of them at a point in their intersection lies in the other.* [**E3, 4.**]

9. **If a straight line is perpendicular to a plane, every plane passed through the line is perpendicular to the first plane.** [**E9a.**]
(**Corollary to 8.**)

10. **If two intersecting planes are each perpendicular to a third plane, their intersection is also perpendicular to that plane.** [**E10.**]
(**Corollary to 8.**)

*11. Through a given straight line oblique to a plane, one and only one plane can be passed perpendicular to the given plane.* [**E11.**]

12. The acute angle which a straight line makes with its own projection on a plane is the least angle which it makes with any line of the plane. [**O13.**]

13. Two right prisms are congruent if they have congruent bases and equal altitudes. [**B1.**]

*14. If parallel planes cut all the lateral edges of a pyramid, or a prism, the sections are similar polygons; in a prism, the sections are congruent; in a pyramid, their areas are proportional to the squares of their distances from the vertex.* [**M1.**]

**(See II, 13.)**

**15. Every section of a circular cone made by a plane parallel to its base is a circle, the center of which is the intersection of the plane with the axis. [M5.]**

**16. Parallel sections of a cylindrical surface are congruent. [M3.]**

## IV. Spheres.

**1. Every section of a sphere made by a plane is a circle. [M10.]**

**(Several corollaries may be added.)**

**2. The intersection of two spheres is a circle whose axis is the line of centers. [H4.]**

*3. The shortest path on a sphere between any two points on it is the minor arc of the great circle which joins them.* [**O14.**]

4. A plane is tangent to a sphere if and only if it is perpendicular to a radius at its extremity. [**M11, 12, 13.**]

**5. A straight line tangent to a circle of a sphere lies in a plane tangent to the sphere at the point of contact. [*]**

6. The distances of all points of a circle on a sphere from its poles are equal. [**H1.**]

7. A point on the surface of a sphere, which is at the distance of a quadrant from each of two other points, not the extremities of a diameter, is the pole of the great circle passing through these points. [**H3.**]

**8. A sphere may be inscribed in or circumscribed about any given tetrahedron. [H5.]**

*9. A spherical angle is measured by the arc of a great circle described from its vertex as a pole and included between its sides (produced if necessary).* [**K5.**]

## V. Spherical Triangles and Polygons.

**(Every theorem stated here may also be stated as a theorem on polyhedral angles.)**

1. Each side of a spherical triangle is less than the sum of the other two sides. [**O11 (b). See also (a).**]

**2. The sum of the sides of a spherical polygon is less than 360°. [O12 (b). See also (a).]**

**3. The sum of the angles of a spherical triangle is greater than 180° and less than 540°. [K8.]**

*4. If A'B'C' is the polar triangle of ABC, then, reciprocally, ABC is the polar of A'B'C'.* [**K6.**]

*5. In two polar triangles each angle of the one is the supplement of the opposite side in the other.* [**K7.**]

6. Vertical spherical triangles are symmetrical and equivalent. [**C8, H6.**]

**7. Two triangles[47] on the same sphere are either congruent or symmetrical if**

| | | | |
|---|---|---|---|
| $a = a'$ | $b = b'$ | $c = c'$ | [**B2 (b). See also (a).**] |
| or $a = a'$ | $b = b'$ | $C = C'$ | [**B3 (b). See also (a).**] |
| or $a = a'$ | $B = B'$ | $C = C'$ | [**B4 (b). See also (a).**] |
| or $A = A'$ | $B = B'$ | $C = C'$ | [**B5 (b). See also (a).**] |

8. Either of the equations $a = b$, $A = B$ is a consequence of the other. [*]

## VI. Mensuration.

(The relation between the areas and volumes of similar solids may be treated as corollaries in individual cases. See preface, article 3. It is understood that certain statements concerning limits may be assumed either explicitly or implicitly; these are not stated as theorems. See Q3, 4, 9, R9, 10.)

**1. An oblique prism is equivalent to a right prism whose base is a right section of the oblique prism and whose altitude is a lateral edge of the oblique prism. [C4.]**

2. A plane passed through two diagonally opposite edges of a parallelepiped divides it into two equivalent triangular prisms. [**C6.**]

*3. The lateral area of a prism is the product of a lateral edge and the perimeter of a right section.* [**Q1.**]

(Corollary of Plane Geometry.)

*4. The lateral area of a regular pyramid is one-half the product of the slant height and the perimeter of the base.* [**Q2.**]

(Corollary of Plane Geometry.)

*5. The lateral area of a right circular cylinder is the product of the altitude and the circumference of the base; i. e.,* $s=2\pi rh$. [**Q5.**]

*6. The lateral area of a right circular cone is one-half the product of the slant height and the circumference of the base; i. e.,* $s = \pi rl$. [**Q6.**]

7. The lateral area of a frustum of a regular pyramid is one half the product of the slant height and the sum of the perimeters of the bases. [**Q7.**]

8. The lateral area of a frustum of a right circular cone is one half the product of the slant height and the sum of the circumferences of the bases. [**Q8 (a).**]

---

[47] The same notation is used as in the plane triangles.

9. The area of a zone is the product of its altitude and the circumference of a great circle; i. e., $s = 2\pi rh$. [Q10.]

(Lemma for 10 below.)

**10. The area of a sphere is the product of its diameter and the circumference of a great circle; i. e., $s = 4\pi r^2$.** [Q11.]

11. The area of a lune is to the surface of a sphere as the angle of the lune is to 360°. [Q12.]

12. The area of a spherical triangle is to the area of the sphere as its spherical excess is to 720°. [Q13.]

**13. The volume of a rectangular parallelepiped is the product of its three dimensions.** [R1, 2, 3.]

(This may be taken as a definition.)

14. *The volume of any parallelepiped is the product of its base and altitude.* [C5, R4.]

15. *The volume of any prism is the product of its base and its altitude.* [R5, 6.]

16. *The volume of any pyramid is one third the product of its base and its altitude.* [C7, R7, 8.]

17. *The volume of a circular cylinder is the product of its base and its altitude, i. e., $v = \pi r^2 h$.* [R11.]

18. *The volume of a circular cone is one third the product of its base and its altitude; i. e., $v = \frac{1}{3}\pi r^2 h$.* [R12.]

19. The volume of a spherical sector is one third the product of the radius and the zone which is its base; i. e., $v = \frac{2}{3}\pi r^2 h$. [R15.]

20. *The volume of a sphere is one third the product of its radius and its area; i. e., $v = \frac{4}{3}\pi r^3$.* [R16.]

(The wording suggests a proof, but that proof is by no means prescribed. The wording is convenient, the proof may even preferably follow 21 below.)

The following two theorems, while not thought by the committee to be indispensable, offer both for student and teacher an outlook for that larger view of geometry and of mathematics as a whole which is very desirable. They forecast important principles in future mathematical courses; they are capable of the most practical direct applications; they offer a possibility of organizing and retaining the important mensuration formulæ given above.

21. *If two solids contained between the same parallel planes are such that their sections by a plane parallel to those planes are equal in area, the two solids have the same volume.* [*]

("Cavalieri's Theorem." Formal proof should not be given.)

22. The volume of any sphere, cone, cylinder, pyramid, or prism, or of any frustum of one of these solids intercepted by two parallel planes, is given by the formula $v = \frac{1}{6} h(t + 4m + b)$, where $t$ is the area of the upper base, $b$ that of the lower base, $m$ that of a base midway between the two,

and where $h$ is the perpendicular distance between the two parallel planes. [**See R13, 14.**]

(This formula also applies to any so-called prismatoid; it is conveniently useful in practical affairs. It should not be proved for the general case, but each separate solid mentioned above, numbers 1 to 20, can be shown to conform to this rule, by a direct check.)

## *CONCLUSION.*

It should be said that the members of the committee are not entirely agreed as to certain minor details of this report. For example, some would place among the exercises certain propositions now in small type; others would prefer to put some theorems in black faced type which are now in italics; others would prefer three types of propositions instead of four; and some would modify certain postulates and would consider as postulates or as propositions to be demonstrated certain theorems included in the list of those requiring only informal proof. The committee does not regard these minor matters of any great consequence, and therefore wishes to be considered as approving the spirit and general tenor of the report, rather than as giving individual sanction to all such details.

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Technical High School, Cleveland, Ohio.
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Teachers College, Columbia University, New York City.
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Polytechnic Preparatory School, Brooklyn, N. Y.
Mabel Sykes,
Bowen High School, Chicago, Ill.

# THE A B C OF AEROPLANE MECHANICS.[1]

By Harvey N. Davis,
*Harvard University.*

The rapid development of aviation, within ten years, from a dream to an accomplished fact is one of the most spectacular events in the history of applied science. As a result, one finds to-day a widespread and, on the whole, most intelligent interest in aviation among all classes and ages in the community. This offers a valuable opportunity to teachers of physics, for if they will inform themselves on even the simplest of the mechanical aspects of aëroplanes, they will find a wealth of interesting material for the enlivenment of instruction in mechanics. For those who read German, an admirable source of information for this purpose is an article by H. Jansen on page 329 of the 1910 volume of the "Zeitschrift für den Physikalischen und Chemischen Unterricht." Another useful paper, by Major Squier of the U. S. Signal Corps, was presented to the American Society of Mechanical Engineers in December, 1908, and can be obtained for a nominal price from the secretary. Books on the subject are numerous, but are usually either too technical or not technical at all. To such as are without access to any of these sources, the following brief summary of some very elementary material may be useful. It will be presented under the four heads of support and propulsion, fore and aft stability and control, lateral stability and control, and horizontal steering. Some of the fundamental theorems of mechanics of which the aëroplane affords illustrations are mentioned in italics.

1. Support and propulsion.

The simplest possible aëroplane is a kite. If a kite is presented to the wind at an inclination that is neither horizontal nor vertical,

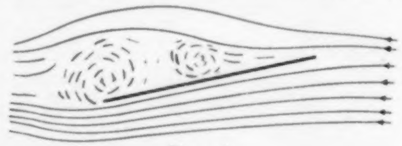

Fig. 1.

the air, in order to get by, must be somewhat diverted downward, as in Fig. 1, and will therefore be somewhat compressed below the kite, so that the pressure on its lower face will be greater than the average pressure in the neighborhood. Above and be-

[1] Read at the spring meeting of the Eastern Association of Physics Teachers.

hind the kite there will be more than the normal amount of room available for the passing air, and therefore expansion and diminished pressure. Each square inch of the kite's surface has, therefore, unequal pressures on its two faces, and is urged upward by a force perpendicular to them; there is also a small tangential force toward the rear due to friction between the surfaces and the passing current of air. All of the forces perpendicular to the kite have at any instant a resultant, also perpendicular to the kite, acting at a perfectly definite point in its surface which may be called the center of pressure (*composition of parallel forces*). Similarly, all the tangential forces have a resultant which can be thought of as acting at the same point (*a force can be displaced along its line of action*). The whole effect of the air on the kite is equivalent to a single force at the center of pressure pointing somewhat behind the normal to the surface at that point (*composition of concurrent forces*). This is the force R in Fig. 2. If the center of pressure happens to coincide with the center of gravity, as in Fig. 3, the weight of the kite, W, acting at the same point, gives, with R, a resultant pointing still more behind the normal.

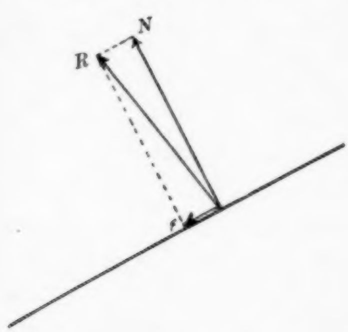


FIG. 2.

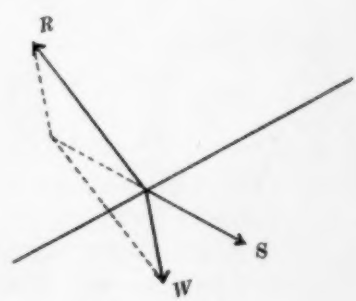


FIG. 3.

If another force, S, equal and opposite to this resultant, can be applied at this point, the kite will be in equilibrium (*equilibrium of a particle*). Such a force is exerted by the kite string. If the kite string is cut away and an engine installed whose weight is equal to the downward component of the force S, and whose "drive" is equal to the forward component of the force S (*resolution of forces*), the kite becomes an aëroplane able to hold its own against the wind that supported the kite, or to make progress against a wind of less velocity.

If the center of pressure does not coincide with the center of

gravity, as in Fig. 4, the force S should be applied at a point different from either the center of pressure or the center of gravity (*three forces, to hold a rigid body in equilibrium, must be concurrent*).

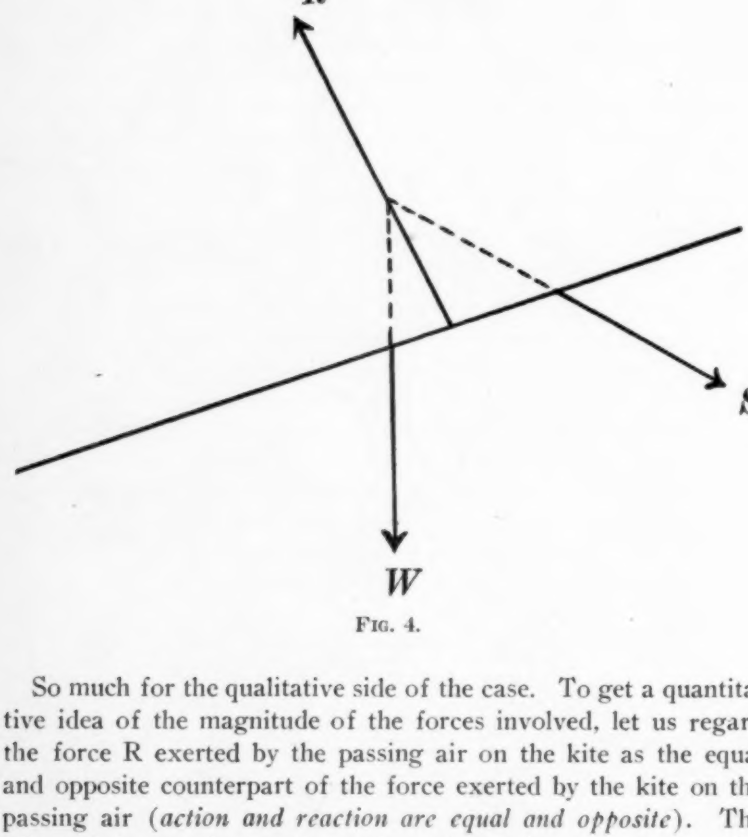


FIG. 4.

So much for the qualitative side of the case. To get a quantitative idea of the magnitude of the forces involved, let us regard the force R exerted by the passing air on the kite as the equal and opposite counterpart of the force exerted by the kite on the passing air (*action and reaction are equal and opposite*). The effect of the reaction on the air is to change its momentum. The fundamental equation (*principle of momentum*) is—

$$Pt = M \triangle V,$$

$$\text{or} \quad P = \frac{M}{t} \triangle V.$$

In this equation, $M/t$ is the mass of air affected in each second. $\triangle V$ represents the average change in its velocity during the second. This change may be in part a change in the speed of the air affected, but it is much more a change in the direction of its motion. It is difficult to get an exact value for $\triangle V$ by any elementary reasoning, but it is both reasonable and experimentally

verifiable that it is proportional to the original velocity, V, of the wind itself.

$$\Delta V = k_1 V.$$

The mass of air affected in each second is also obviously proportional to V. It is also proportional to the density of the air, D, and (within the limits of size common in aëroplanes) to the area, A, of the supporting surface.

Therefore,

$$\frac{M}{t} = k_2 VDA.$$

$$\textbf{Therefore} \quad R = kDV^2A.$$

The value of the factor of proportionality, *k*, in this equation depends on a number of things. Among them is the shape of the plane. If it is a rectangle and is presented end forward, more air will spill around its sides than if it is presented long side forward, and *k* will be larger in the latter case. This is one advantage which Hargrave or box kites have over the older form. Another consideration is the fore and aft curvature of the surface. It is found that *k* is larger if the surface is slightly concave downward or shaped like an elongated S. Another most important consideration is the angle, $\theta$, which the chord of the surface makes with the horizontal, the so-called angle of presentation. The greater this angle, the greater the value of *k*—but the exact form of the relation is still uncertain. The simplest possible theory, proposed by Newton, leads to the law—

$$k_\theta = k_{90} \sin^2\theta$$

but experiments do not verify this. Many other expressions involving $\theta$ have been suggested, of which $\sin\theta$ is probably as good as any for practical use; a list of them can be found in Jansen's paper; all that is necessary for the present purpose is the fact that *k* increases with $\theta$.

In any case, for a given surface presented at a given angle in air of given density, R varies as the product $V^2A$. The horizontal component of R is called the head resistance and varies as $V^2A$. The vertical component of R is called the lift and it also varies as $V^2A$. The first is equal and opposite to the propelling force which the engine must exert. The latter is equal and opposite to the maximum allowable total weight.

Two conclusions can be drawn:

A. The speed law—the force necessary to propel a given aëroplane with supporting surfaces of invariable size, at different

speeds, varies as the square of the speed. Therefore the horsepower required varies as the cube of the speed (*definitions of work and power*). It required 20 H. P. to drive the 1905 experimental Wright machine at a speed of 38 miles an hour. It would have taken 160 H. P. to drive it at the 76 miles an hour which will be required of a successful 1911 racing machine. On the other hand, if the 925 pounds of the 1905 model could just be supported at the slower speed, 7,400 pounds could be supported at the higher speed, which would leave over three tons of extra lift available for supporting the larger engine.

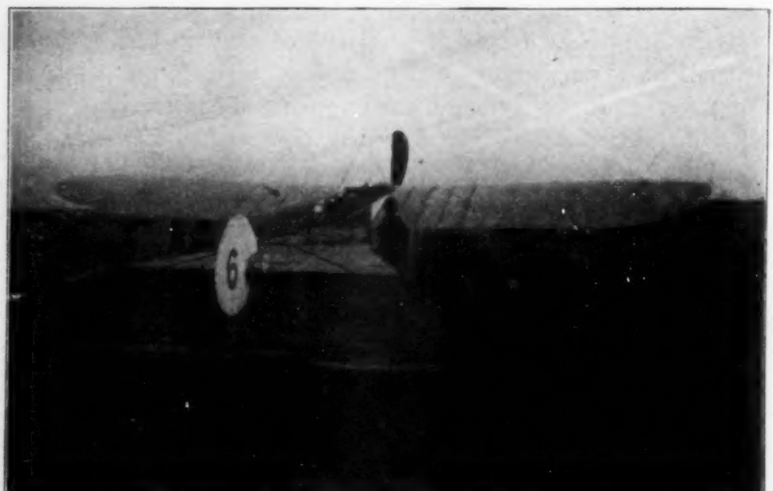


**Figure 5. A Bleriot Monoplane. This is the passenger carrying machine used by De Lesseps at Belmont Park. Notice the unusually large fixed horizontal surface in the tail, for automatic fore and aft stability, and the two kinds of rudders. Notice also the slight diedral angle formed by the wings, for automatic lateral stability. Lateral control is secured by rotating the whole of each wing about a transverse axis near its front edge.**

B. The law of reefing.—If an aëroplane of invariable weight is to be driven at various speeds, with bare support at each speed, the exposed area of its wings should be progressively diminished as the speed increases, in such a way as to keep the product $V^2A$ constant. In other words, its wings should be reefed like the sails of a boat. It will be easy for the reader to verify for himself that under these circumstances the force necessary to keep such an aëroplane in motion is independent of the speed, while the horsepower required increases only as the first power of the

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**[1]I am indebted to Mr. E. C. Brown of the Harvard Aëronautical Society for these pictures.**

speed. If the wings of the 1905 Wright machine could have been reefed to one quarter of their original size (and if the head resistance of the framework and operator could have been decreased in the same proportion) it could have been driven at 76 miles an hour on only 40 H. P. This was very nearly what was done in designing the "Baby Wright" racer that was so unfortunately wrecked on the morning of the Gordon Bennett Cup Race last October. Its wing spread was somewhat more than a quarter of that of the 1905 model because of the increased weight of the engine and the impossibility of reefing the operator's body, but its horsepower was only about 60 and yet it is believed to have reached 70 miles an hour.

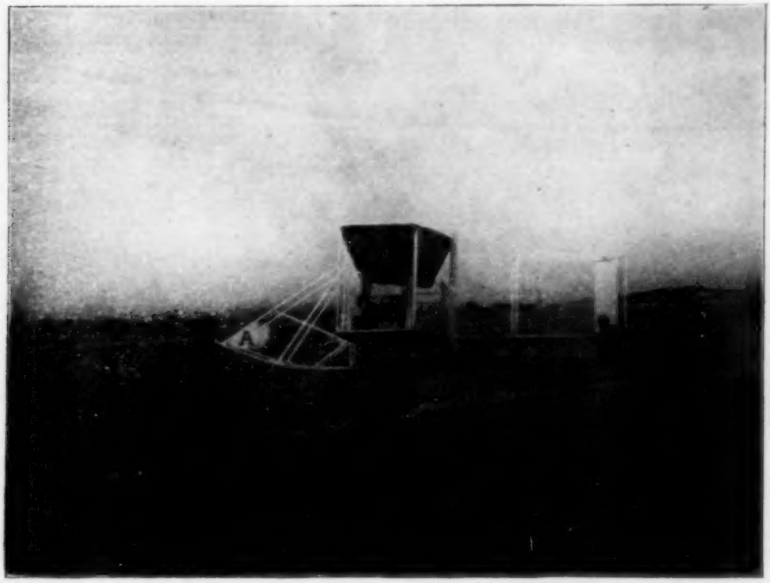

Figure 6. A WRIGHT BIPLANE. This is a "headless" model used by the Wright flyers at Squantum. The two rudders are in the rear; the horizontal rudder, when at rest, acts also as an automatic stabilizer. Lateral control is secured by warping the wings, so as to raise or lower their outer rear corners. The small semicircular sails on which "A" is painted are rudimentary keels.

The realization in practice of this principle of reefing, while a machine is in the air, is probably the most important step soon to be taken in the development of the aëroplane.

2. Fore and aft stability.

This can be secured either through the conscious skill of the operator or, in part at least, automatically. It can be secured

automatically by mounting a small fixed horizontal surface at some distance from the main planes. This surface cannot be in front, as the equilibrium in the horizontal position in the face of the onrushing current of air would be unstable. If the surface is behind, the equilibrium is stable. The principle is that of a weathercock. The early Wright machines had no horizontal tail, but depended entirely on the skill of the operator. Their new models agree with most other makes of aëroplanes in having one.

For the conscious control of fore and aft stability, and for steering the aëroplane up and down, a movable horizontal rudder is provided. If this is in front and is so tipped as to point the machine upward, not only is the lift of the main surfaces increased so as to lift the machine, but the rudder adds some lift of its own. If it is behind and is so tipped as to point the

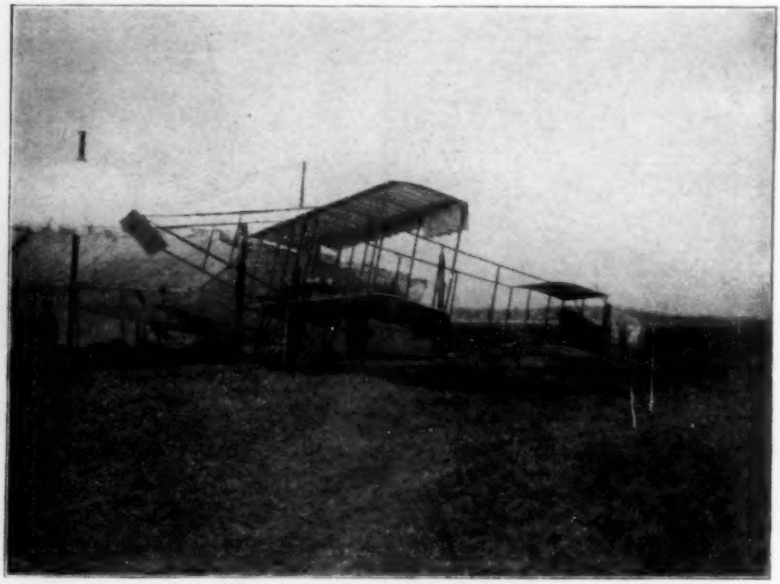

**Figure 7. A Farman Biplane. This is the machine used by Graham-White at Squantum. Notice the large fixed horizontal surfaces in the tail, and the elevator in front. Notice also the rudders in the rear. Lateral control is secured by four movable wing tips, which are hanging loose in the picture.**

machine upward, its own small effect is wholly downward, and the rise of the machine is therefore less rapid (*theorem of the motion of the center of mass of a system*). It is therefore theoretically preferable to have the elevator in front, but, for structural reasons, rear elevators are very often used.

3. Lateral stability.

Here again either conscious or automatic control may be sought, but the automatic lateral stability so far attainable is much less valuable than in the fore and aft case. The only simple principle of any importance is one embodied in a few biplanes and in most monoplanes. It is very noticable, for example, in the Antoinette monoplane, the wings being several feet higher at their tips than at their points of attachment, forming a very blunt V. The effect of this in automatically preserving lateral equilibrium can be observed by anyone who will bend a 3x5 card through its center parallel to its shorter edge until its halves make an angle of about 120° with each other, and will then drop it from various initial orientations. The mechanical principle in

Figure 8. A Burgess-Curtis Biplane. This machine was flown by Curtis at Squantum. Like the Farman it has a fixed horizontal stabilizer behind and a movable elevator in front. Lateral control is secured by ailerons or separate wings, between the main supporting surfaces.

action is the principle of moments, both the lever arm and the air resistance itself being greater on the side that is too low.

Conscious lateral control is secured by some device that enables the lift of each wing to be varied independently. In the Wright machine this is accomplished by warping the whole of each wing, so as to raise or lower its outer rear corner. This alters the average angle of presentation of each wing in such a way as to

increase the lift on the low side and decrease it on the high side. One incidental effect is that the head resistance is also increased on the low side and decreased on the high side and this would tend to swing the machine off its course if the rudder were not so connected as to be automatically turned in the compensating direction at the same time. This is the substance of the famous Wright patent.

Other similar devices are used by other makers. In the Bleriot and Antoinette monoplanes, for instance, each wing is rigid, but can be slightly rotated as a whole around a horizontal axis so as to change its lift. In the Farman and some of the Curtiss models most of each wing is rigid and immovable, but there is a small tip hinged to the rest, which can be moved. In other Curtiss models, and in Burgess-Curtis machines, small movable wings are mounted between the main surfaces near their tips. In the Pfitzner monoplane, not yet a commercial success, arrangements are provided for increasing the lift of the low side wing by actually changing its exposed area; this device is a promising forerunner of true reefing.

4. Horizontal steering.

It is easy to change the orientation of an aëroplane in motion by turning a vertical rudder, but this is not enough to make an aëroplane negotiate a curve successfully. Something analogous to the keel of a boat must be provided to exert on the aëroplane a sidewise force of sufficient magnitude to pull its center of mass around the curve (*centripetal force*). This can be accomplished in either of two ways. (a) In the Voisin biplanes (and in some others) large vertical surfaces between the main planes (as in a box kite) give a sidewise thrust whenever the head of the machine is pointed to one side. (b) The mechanism for lateral control can be consciously used by the operator to bank the whole machine like a railroad train on a curve, until the normal pressure on the under side of the main supporting surface yields a horizontal component large enough to produce the desired centripetal acceleration. This is by far the commoner practice, and it is the basis of some of the most beautiful aërial manœuvers that are executed, such as the famous *vol planes* of the Wright flyers. Many interesting problems can be based on the frequently published particulars of these feats. For example, Brookins is said to have made a complete circle in six seconds in a machine that must have been going at about 40 miles an hour; at what angle did he probably bank his planes?

In the future of aëroplaning there are many problems that will probably turn out to have scientific aspects almost as simple as those already described. The most important of these is the realization of the principle of reefing. Next is the attainment of a much greater measure of automatic lateral stability. Next is the provision of effective "air brakes" for producing powerful retarding forces while an aëroplane is still in flight, so that it can land on the roof of a modern office building. Powerful accelerating forces are also needed to enable it to start safely from such a perch; perhaps these can be provided for by temporarily reducing the head resistance through reefing. And if in addition to these, someone can provide for such a division of the power plant as will enable one half of it to drive the aëroplane at half its normal speed under sufficient sail area while the other half of the power plant is being adjusted in midair, the day of the aëroplane will have come for good.

## COAL LANDS AWAIT VALUATION.

### Nearly 80,000,000 Acres Yet to be Classified by United States Geological Survey.

The government has a gigantic task in hand in the classification and valuation of its coal lands in the West. These lands are probably the nation's greatest direct asset, not even excepting the millions of horsepower latent in the rivers on the public lands. During the last two years the United States Geological Survey has examined in great geologic detail over 15,000,000 acres of the Western coal lands, and in the month of March it classified and appraised 1,220,748 acres, with a valuation of $37,971,740. There still remains withdrawn, however, 78,152,808 acres awaiting classification and valuation. The following table shows the states in which these lands are situated:

COAL LANDS WITHDRAWN AND AWAITING CLASSIFICATION APRIL 1, 1911.

| | Acres. |
|---|---|
| Arizona | 118,718 |
| California | 239,903 |
| Colorado | 5,866,763 |
| Idaho | 8,266,509 |
| Montana | 21,393,613 |
| New Mexico | 2,532,038 |
| Nevada | 92,141 |
| North Dakota | 18,215,384 |
| Oregon | 3,711 |
| South Dakota | 2,375,263 |
| Utah | 6,128,923 |
| Washington | 2,207,967 |
| Wyoming | 10,711,875 |
| Total area | 78,152,808 |

## SIMPLE DEMONSTRATION OF COLOR MIXTURES.[1]

By H. Teike,
*Strassburg, Germany.*

The mixture of colored light may be demonstrated in a simple manner with the aid of colored mirrors. Three ordinary mirrors, 12x15 cm., are mounted one above the other on an adjusting stand so that they may be held in any position (Fig. 1). Each

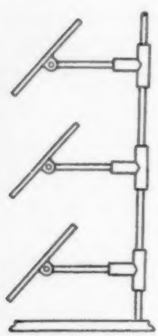

Fig. 1.

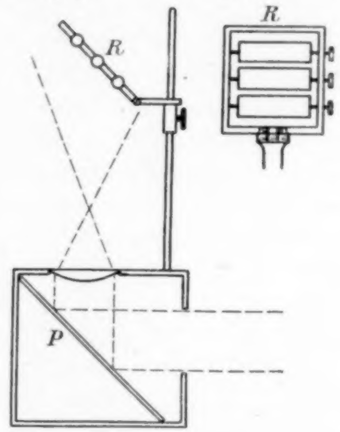


Fig. 2.

mirror is covered with a layer of colored gelatine, so that they show the fundamental colors, red, green, and blue (the usual filter colors in photography). The apparatus is placed in a sunlighted room so that the reflections from the mirrors fall on a conveniently located screen. By adjusting the mirrors any two or all three reflections may be made to cover each other so that one gets by the mixture—

blue and green............................light blue
red and blue..................................violet
red and green...................a pronounced yellow
red, green, and blue....................a clear white

The mirrors may be easily removed from the frames to be replaced by those of other colors in case different mixtures are to be demonstrated.

If several reflections are united on the screen, an object held before one of the mirrors, for example, a pencil, throws a shadow, the color of which is easily explained when one remembers that

[1] Zeitschrift für den physikalischen und chemischen Unterricht. März, 1910.

the object retains one color which of course is wanting in the shadow picture.

In the reflection from a single mirror one obtains shadows in the complementary colors (effect of contrast). For this experiment the screen must be sufficiently illuminated with white light as well as with the colored mirrors. This can be easily accomplished by the reflection from an ordinary mirror or from a projecting lantern.

For these experiments artificial light can be used as well, as, for instance, that from an ordinary stereopticon. In this case the mirrors are placed in the luminous cone of the lantern. If an apparatus for the projection of horizontal objects is at hand, it is best to use, instead of the upper mirror, a small frame with three colored mirrors (Fig. 2). The objective of the lantern must of course be removed during the experiment.

The stand with three movable mirrors in the fundamental colors, red, green, and blue, as well as the frame with the three small mirrors for the stereopticon may be had of Herr. F. Mayer, Strassburg i. e. Krämesgasse 10.

## THE CARTESIAN DIVER.

By Philip Fitch,
*North Side High School, Denver.*

A number of interesting and instructive experiments can be performed with the Cartesian Diver. In the experiments described below, the writer used a medicine vial having a length of one and one half inches and a diameter of one half inch, and a small hydrometer jar six inches high and having a diameter of one and one half inches.

The jar was filled to within one half inch of the top with water at 4° C. Then the amount of water in the vial was adjusted so that it just sank when inverted in the jar. (This adjustment can be easily accomplished after a little practice.) Water at 70° C. was then carefully introduced into the jar and enough water, at the same time, removed to keep the surface at the same level. This operation was continued until the water in the jar reached a temperature sufficiently high to cause the air in the vial to expand enough so that the weight of the displaced water was slightly greater than that of the vial and the air contained in it. The vial rose to the top. It could then be forced to sink again by pressing on the top of the jar with the fleshy part of the hand.

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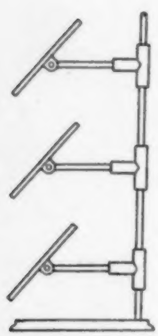

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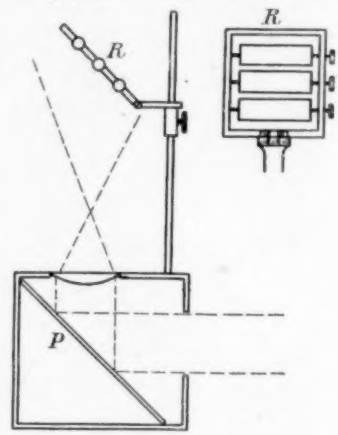


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In another experiment, water was used at the temperature of the room and placed in a tumbler to within one fourth of an inch from the top. The vial was again carefully introduced, so that it just sank to the bottom. The tumbler was then placed under the receiver of an air pump and enough air exhausted to cause the vial to float. Upon allowing the air to slowly re-enter the receiver, the vial lowered slightly but did not sink, being

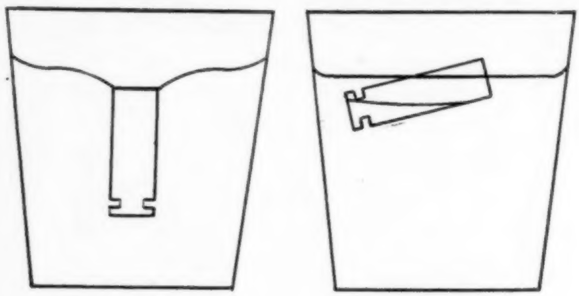


FIG. 1. FIG. 2.

supported by the surface tension of the water. The surface of the water took the shape shown in Fig. 1. After giving the receiver a slight jar with the hand, the vial sank to the bottom. The air was again exhausted, but this time the pumping was carried on until the vial took the position shown in Fig. 2 and two small bubbles of air escaped from it. Upon allowing the air to slowly reënter the receiver, the vial slowly assumed a vertical position, lowered to the same position as in the previous experiment, and then suddenly sank to the bottom.

## A SIMPLE REFLECTING GALVANOMETER.

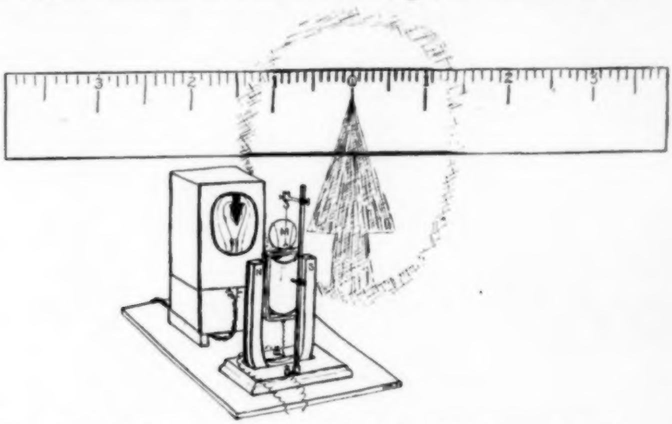
By J. M. ARTHUR,
*Tome School, Port Deposit, Md.*

The reflecting galvanometer shown in the illustration embodies one or two ideas that turn the usual complicated instrument into a very simple machine.

After testing apertures of different sizes and shapes with lenses of various diameters and focal lengths, I found that the illumination and definition of the index-spot were so poor that I tried the arrangement as illustrated.

First, the little half inch plane mirror on the coil was removed and in its place was mounted, by a simple clamp of bent wire, the inch and a half *concave* glass mirror, M. Then a large opening

with an inverted arrow point for an index was placed in front of the light. The light was adjusted in a moment so that it and the scale were at conjugate focii of the mirror. A large and fairly clear cut image of the pointer fell on the scale, as shown. The scale which I have is six feet long with its smallest divisions



an inch apart and is mounted about eight feet from the galvanometer. The lamp is not more than six inches from the mirror. Even though the heavier mirror retards the coil somewhat, a common six-inch bar magnet, when inserted in a small coil, gives a throw of three or four inches; and by repeated insertions, properly timed, the index can be made to pass the whole length of the scale.

The parts need not be permanently mounted. The lamp and galvanometer may be placed on the lecture table and by simply tilting the mirror slightly backward the index may be thrown on a scale drawn on the blackboard in the rear of the table. In practice, of course, a light-screen is placed back of the galvanometer.

The economy of space, the absence of lenses, the plentiful illumination, and the ease of adjustment, make the instrument a very satisfactory piece of apparatus.

## THE "WALKING" CURE.

**Doctors are of one mind in advocating walking as one of the best means of keeping the human machine in good working order, and one of New York's foremost medical authorities went as far as to say, in a lecture to young men studying for the profession, that if every adult could be persuaded to make a conscientious habit of walking five miles every day, there would be such a prompt and general improvement in health that doctors would soon have to be looking for other ways of making a living. —*Physical Culture for May.***

# A METHOD OF CARBON DIOXIDE ANALYSIS.[1]

By F. C. Irvin,
*Central High School, Detroit,*

The customary method for determining the composition of carbon dioxide is by burning a weighed amount of carbon and absorbing and weighing the carbon dioxide produced. After several careful trials of this method we were forced to the conclusion that our equipment and limited periods made the experiment impracticable for fairly large classes. Believing, however, that some form of carbon dioxide analysis is a desirable experiment, I was particularly pleased to find a workable method for high school students described by Professor J. T. Stoddard.

It is an adaptation of his method to a limited equipment that this paper describes. As an exact title for the experiment we may state, "To determine the amounts of carbon monoxide and oxygen in carbon dioxide."

A brief outline of the experiment is as follows:

1st. Carbon dioxide is led over heated zinc dust which reduces it to carbon monoxide, the zinc forming zinc oxide. The carbon monoxide is collected over sodium hydroxide and the volume measured.

2nd. The apparatus necessary consists of an open ignition tube about 30 cms. long and 1.5 cms. diameter, delivery tube and 100 c.c. to 150 c.c. gas measuring tube; a Kipp apparatus for carbon dioxide, wash bottle and calcium chloride drying tube.

3rd. To set up the apparatus and work the experiment, we place a loose wad of asbestos near one end of the ignition tube, add about 5 g. of zinc dust, and secure it in position by another piece of asbestos. Dry the zinc dust by gentle heating in a current of air, and when cool, weigh.

Set up a Kipp apparatus for carbon dioxide, provide it with a wash bottle, and drying tube and connect the ignition tube to the drving tube. Fill the 150 c.c gas tube with a 20 per cent solution of sodium hydroxide and invert it over a dish of the same supporting the tube with a clamp. Arrange the delivery tube from ignition tube so that it dips under the mouth of the gas tube. The apparatus is now ready for operation.

[1]Read before the Physical and Chemical Conference of the Michigan Schoolmasters' Club, March 31st.

We must first expel air from the apparatus. This is accomplished by removing the mouth of the delivery tube from the gas tube and allowing carbon dioxide to pass freely through the apparatus for one minute. Next bring the mouth of the delivery tube under the gas tube and heat the zinc dust in the ignition tube. Regulate the current of carbon dioxide so that it passes through the wash bottle at the rate of 2 or 3 bubbles per second. When the volume of gas (carbon monoxide) in the gas tube measures abut 100 c.c., remove the burner but continue the current of carbon dioxide for 3 or 4 minutes to drive all the carbon monoxide into the gas tube. Allow the gas to stand over the sodium hydroxide for a short time to absorb all the carbon dioxide. This may be determined by noting when the volume becomes constant. Then transfer the gas tube to a cylinder of water, lower it until the level of the liquids is the same inside and out, and read the volume of carbon monoxide. Also obtain the barometer reading, the temperature of the gas and the aqueous tension. When the ignition tube is cool, weigh again. The gain in weight is the weight of oxygen removed from the carbon dioxide. We next correct the volume of gas to standard conditions, and assuming that one liter of carbon monoxide weighs 1.25 g., we compute the weight of carbon monoxide.

The above outline is essentially that given by Professor Stoddard in his excellent manual, "Quantitative Experiments in Chemistry."

If we supplement the directions with a tabular form for recording results, we have found that the students obtain a clearer comprehension of the experiment. We have used the following with good results.

| 1st wt. of tube | 2d wt. of tube | wt. of O | barometer | Temp. of Gas | Aq's tens | Obs'd vol. CO | Corr'd vol. CO | wt. of CO | Per cent of CO |
|---|---|---|---|---|---|---|---|---|---|

That the experiment is workable by high school students, will be granted, I believe by an inspection of the following chart, which shows the results obtained in one of our sections in the work of last semester.

It will be observed that the students collect approximately 100 c. c. of carbon monoxide instead of 180 c.c. to 200 c.c. as Professor Stoddard directs. While the larger volume will en-

| Observed Vol. CO | Corrected Vol. CO | Weight of CO | Determined wt. of O | Computed wt. of O |
|---|---|---|---|---|
| 76.0 c.c. | 66.2 c.c. | 0.083 g. | 0.04 g. | 0.047 g. |
| 103.1 | 90.2 | 0.113 | 0.06 | 0.064 |
| 104.0 | 91.0 | 0.114 | 0.07 | 0.065 |
| 45.0 | 38.2 | 0.048 | 0.03 | 0.027 |
| 100.3 | 87.7 | 0.112 | 0.06 | 0.063 |
| 101.0 | 88.5 | 0.1125 | 0.06 | 0.064 |
| 95.0 | 83.0 | 0.104 | 0.06 | 0.060 |
| 110.0 | 94.9 | 0.119 | 0.07 | 0.068 |
| 112.5 | 97.0 | 0.1225 | 0.07 | 0.070 |
| 107.8 | 93.0 | 0.117 | 0.07 | 0.067 |

able the students to obtain more accurate results, we favor the smaller volume because the students can then complete the experiment during the regular laboratory period. Futhermore, many schools are not supplied with the larger gas tubes. Also the fourth column of the table indicates that the students were working with a balance sensitive to centigrams only. With a balance weighing to milligrams, more satisfactory results, of course, may be obtained. Admitting the handicap of a poor balance, however, I think the experiment shows the following advantages:

1. Comparative simplicity of the apparatus.
2. No danger of loss of carbon monoxide.
3. Rapidity with which any carbon dioxide is absorbed by the sodium hydroxide.
4. The average student is intensely interested in this form of quantitative determination. He obtains the barometer reading, temperature, and aqueous tension, important features in themselves, and finally, with reasonable care he may expect fairly accurate results.

The experiment follows the preparation and qualitative study

of carbon dioxide and also carbon monoxide in the laboratory, so that the student is familiar with the properties of these gases before taking up the quantitative determination.

A final point of excellence in the experiment is the readiness with which zinc dust reduces carbon dioxide. As the examples of easily measurable and direct oxidation and reduction are rather few, this may be one of the most important features of the experiment. I believe that an experiment of this nature is a very valuable part of the laboratory course of the first year's work in chemistry.

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## SOLIDIFIED GASOLINE.

Consul Albert Halstead sends from Birmingham a press description of a chemist's invention for converting gasoline or petrol into a stiff, white jelly. It is effected by adding 1.75 per cent of steatite and alcohol. A high-powered automobile recently made long trips in England successfully using the new jelly, for which the inventor claims an economy of 30 per cent.

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## PETROLEUM FROM ALASKA.

The first shipload of oil from the Katalla (Alaska) wells is expected southward in May. Crews of men are at work erecting three steel tanks in the vicinity of Katalla, and laying eight miles of pipe line to connect the tanks and the wells. Steel tankage to the amount of 102 tons has just reached the ground, in the form of plates and framework. One tank will stand at tidewater, and will contain 30,000 barrels of oil—a good-sized shipload. This reservoir will be 86 ft. in diameter, and will weigh 80 tons. Two other tanks will be erected beside the wells, one of 2,000 barrels and the other of 5,000 barrels capacity.

These Katalla wells—four of which have a daily flow of 2,000 barrels—have had a strange history. The victims of contending interests, their flow has been deliberately choked with old junk of all kinds, in an effort to lessen their value. But now, after half a dozen years, they are to be kept open.

Existence of oil in the Katalla district has been known for 15 years. Indians noticed the seepage and little lakes of the blackish fluid along the coast for 25 miles to the east of the Copper River delta. This belt of territory lies adjoining the great Katalla coal measures that have so long been under contention. Traders and pioneer miners rushed to the Katalla district, and in 1896 and 1897 staked out many of the visible locations. Development went no farther, and the outside world was given the hint that the Katalla properties were valueless commercially. With the last year new ownerships have prevailed and work was begun clearing the debris out of the wells, and one is said to have flowed 720 barrels a day, after it had blown the top of the derrick away in its first rush of pressure. A measured test of the three others, taken with that, is said to have showed the total of 2,100 barrels. In some places oil was found under 1,000 ft., and the deepest producing well in the Katalla belt is only 1,500 ft. deep. —*Mining Science*.

## A BALANCE FOR WEIGHING ONE TEN THOUSANDTH PART OF A MILLIGRAMME.

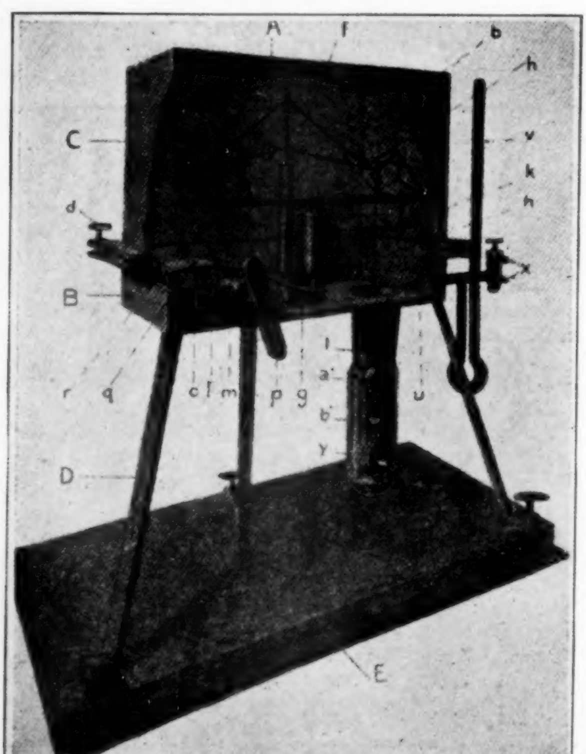
With ordinary chemical balances weights can be determined to within 1/10 milligramme (1/650 grain). This degree of accuracy is quite sufficient for most purposes, but greater precision is required for the solution of the new problems in chemistry which have followed the discovery of radium and other radio-active substances.

We know that radium, uranium, and thorium are subject to slow but sure spontaneous decomposition, and it is not impossible that other elements obey the same law. It is quite impossible to detect such decomposition by means of loss of weight if we employ ordinary chemical balances, for even radium loses less than two millionths of its weight in a day, or 1/1600 of its weight in a year.

Steele and Grant have devised a balance with which a loss of weight as small as 1/10,000 milligramme can be determined with accuracy. This balance, the precision of which can be increased to 1/250,000 milligramme, has already been employed by Ramsay to determine the loss of weight of radio-active substances.

The following description of this marvelously sensitive instrument is quoted from the *Proceedings* of the Royal Society.

The construction of the balance is illustrated by the accompanying engraving. The balance is enclosed in an air-tight metal case, which is supported



by three legs, D, terminating in leveling screws which rest on a marble base, E. The cover of the case, C, is attached to the bottom, B, by means of the screw *d*. The air is drawn from the case through the three-way cock *x*, and the degree of rarefaction is indicated by the vacuum meter *v*. The apparatus is very much smaller than an ordinary balance. The case is about 5 inches long, 4 inches high, and 2½ inches deep. The beam of the balance A is constructed of very slender rods of quartz, arranged in the form of two triangles, and is provided with a knife edge of rock crystal F, which rests on the metal post *b*. The balance pan *b'* is suspended from the right arm, while the left arm carries a quartz bead, which serves as a counterpoise. Especial care was devoted to the mechanism for stopping the oscillation of the balance, without injuring the delicate instrument, affecting the accuracy of the weighing, or admitting air into the case. By turning the handle *p*, the shaft *l* is rotated in the bearing *m*, causing the cam *o* to raise or lower the spring *q* which is hinged at *r*. The movement of the spring is communicated to the vertical cylinder *g* and the horizontal rod *h*, which carries two small triangles of quartz fibers which embrace the arms of the balance.

It would be impossible to use weights of platinum or even of quartz for the exceedingly delicate measurements for which this balance was designed. The substitute for weights is the most original feature of the apparatus. A thin quartz bulb *a'*, filled with air and sealed, is suspended from the right arm of the balance, above the scale pan *b'*, to which a small auxiliary weight *y* can be attached if necessary. This part of the apparatus is contained in the glass tube *t* which projects below the bottom of the metal balance case, to which the mouth of the tube is attached by an air-tight joint. A little phosphoric acid is placed in the bottom of the tube, to absorb every trace of moisture. The effective weight of the sealed quartz bulb *a'* increases as the air pressure inside the balance case and tube diminishes.

An experiment to determine the loss of weight of a radio-active substance is conducted as follows: A little of the substance, not exceeding 1/10 gram (1½ grains) in weight, is placed in the scale pan and balanced by adjusting the weight of the quartz bead carried by the other arm of the balance. The final adjustment can be made with great accuracy by heating the slightly too heavy head in the oxy-hydrogen blow pipe until a sufficient quantity of quartz has been vaporized. The movements of the balance are read by means of a little mirror, attached to the beam (directly under A in the illustration), which reflects the rays of a lamp to a cathetometer. As the substance in the scale pan loses weight the equilibrium is disturbed, and the pan *a'* and quartz bulb *b'* rise. The loss of weight is determined by exhausting air from the balance case until the equilibrium is restored. From the readings of the vacuum meter *v* at the two moments of equilibrium, the loss of weight can be calculated. A change of 1 millimeter in the reading of the vacuum meter corresponds to a change in weight of about 1/76,000 milligramme. As the height of the mercury column can be read accurately to 1/10 millimeter, it is thus theoretically possible to detect a change in weight of a little more than one millionth of a milligramme, but friction and other sources of error reduce the precision attainable in practice to from 1/10,000 to 1/250,000 milligramme.—*Scientific American.*

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"The Rational Geometry of Professor George Bruce Halsted has been translated into French and will be published by the firm Gauthier-Villars."

## WELL-DRILLING LORE.

### United States Geological Survey Issues Report on Methods of Drilling for Water, Oil, and Other Underground Resources.

The deepest well in the United States is near West Elizabeth, Pa. Its bottom is 5,575 feet beneath the surface. The deepest well in the world is in Germany and is 6,572 feet deep. A more remarkable well, perhaps, reaching a depth of 3,600 feet, was drilled for petroleum in western China by primitive methods and by means of such crude appliances as a cable made of twisted strands of rattan.

These facts and much other interesting information concerning underground supplies of water and oil and methods of getting at them are given in a report entitled, "Well-Drilling Methods," by Isaiah Bowman, just published by the United States Geological Survey as Water-Supply Paper 257. The report is comprehensive and well illustrated, covering 130 pages and containing plates and figures, and may be considered one of the semi-popular publications of the Survey.

All rocks contain some water, but some formations, such as granites, carry only an inappreciable amount. Sandstone, on the other hand, has an absorptive capacity of a gallon or more of water per cubic foot of rock and is the best water bearer of the solid rocks. Wells sunk in sandstone are usually drilled and the water derived from that rock is seldom polluted. To those who are contemplating sinking wells or increasing their water supply from underground sources, this report, taken in connection with another report recently published by the Survey, "Underground Waters for Farm Use" (Water-Supply Paper 255), will be of interest.

Water-Supply Paper 257 traces the history of well drilling from its earliest practice in China down to date and contains descriptions of the many methods of drilling now in use in the United States, including not only drilling for water but for oil and other resources. The credit of reducing well drilling to a science belongs to the Chinese, but in this, as in many other things, the Chinese engineers have made but slight improvement during the last century, which has witnessed so remarkable an advance in mechanical development in the United States.

Mr. Bowman regards the use of well casing as the greatest improvement yet devised for oil drilling and notes that holes can now be sunk safely and rapidly to a depth of 5,000 feet. He describes the various tools and rigs required for different kinds of drilling, ranging from those required for the shallow hand-driven well of perhaps twenty feet to those used in drilling wells of maximum depth.

"How can work proceed when a drilling tool is lost in a deep well?" is a question which may often be asked by any person unfamiliar with methods of well drilling. The recovery of lost or broken drilling tools is, in fact, one of the most difficult features of well drilling. Simply to ascertain the nature of an accident which has occurred half a mile below the surface in a hole perhaps only six inches in diameter requires some skill. Many devices have been perfected for capturing lost tools, and the necessity for first learning the shape of the upper end of the tool and the position in which it lies in order to proceed to recover it, has even led to the invention of a small camera which can be lowered to the bottom of the hole to take a photograph showing the conditions. Electricity is, of course, the agent employed in operating the device.

A copy of this report (Water-Supply Paper 257) or a copy of Water-Supply Paper 255, mentioned above, can be procured by applying to the Director of the United States Geological Survey, Washington, D. C.

## AN IMPORTANT PAINT CONSTITUENT.

### United States Geological Survey Issues Report on Production of Barytes.

Barytes, or heavy spar, is an important constituent of mixed paints. It is known as a reinforcing or inert pigment, and in this respect may be classed with gypsum and silica. In the past the overloading of mixed paints with inert pigments has been the main cause of a strong prejudice that has grown in the minds of consumers against ready mixed paints. After very thorough laboratory tests the scientific section of the Paint Manufacturers' Association and the Master Painters' Association of Philadelphia have concluded that although barytes and other inert pigments have no especial value as pigments when used alone, if intelligently used within certain limits they form an important ingredient in a satisfactory mixed paint and that their use in paints in quantities up to 15 per cent is thoroughly justified, producing better paints for general exterior painting.

Mr. Ernest F. Burchard, in an advance chapter from Mineral Resources of the United States for 1909—"The Production of Barytes and Strontium"—states that these investigations have demonstrated the legitimate place of barytes products among pigments, helping to restore confidence in them, and that it is expected that the depression in the trade caused partly through misuse of the material by paint makers and partly through misapprehension on the part of consumers will soon be overcome. The report states that the quantity of crude barytes mined in the United States in 1909 was 58,377 short tons, valued at $198,561.

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## IMPORTANT SCIENTIFIC DISCOVERY.

A discovery in the fundamentals of electrical science, augmenting previous discoveries, has just been made by Dr. Robert A. Millikan, professor of physics in the University of Chicago, and Mr. Harvey Fletcher. By this newest discovery the authors have given complete proof of the correctness of the atomic theory of electricity and have also given a much more satisfactory demonstration than had before been found of the perpetual dance of the molecules of matter.

Last spring Professor Millikan announced the isolation, for the first time in the history of the science, of an individual atmospheric ion—an electrically charged molecule of air—holding it under observation for an indefinite length of time, and making a direct study of its properties.

The newest experiment and its results are described in detail in the following statement of the work carried on at the Ryerson Physical Laboratory:

"In the report which Professor Millikan made last spring of the work which he and Mr. Fletcher had been doing on the isolation of atmospheric ions, it was shown that all electrical charges are built up of elementary atoms of electricity and the value of this ultimate electrical atom was accurately determined. The method consisted in catching atmospheric ions upon minute oil drops floating in the air and measuring the electrical charge which the drops thus acquired. This year the following extensions of this work have been made:

"(1) The act of ionization itself is now being studied, each of the two electrical fragments into which a neutral molecule breaks up being caught upon oil drops at the instant of formation. This study has shown that the act of ionization of a neutral air molecule always consists in the detachment from it of one single elementary charge rather than of two or three such charges.

"(2) By suspending these minute oil drops in rarified gases instead of in air at atmospheric pressure, the authors have been able to make these oil drops partake of the motions of agitation of the molecules to such an extent that they can be seen by any observer to dance violently under the bombardment which they receive from the flying air molecules. By measuring accurately the amount of the motion of agitation of the oil drops and comparing it with the motions which they assume under the influence of an electrical field because of the charge which they carry, the authors have been able to make an exact and certain identification, with the aid of computations made by Mr. Fletcher, of the electrical charge carried by an atmospheric ion (and measured in their preceding work), with the electrical charge carried by univalent ions in solution.

"This work not only furnishes complete proof of the correctness of the atomic theory of electricity, but gives a much more satisfying demonstration than had before been found of the perpetual dance of the molecules of matter."

## MILK AS A FOOD.

Milk is undoubtedly more easily digested than any other food. It is more nearly allied to blood. It is quickly absorbed into the circulation and becomes a part of the tissues of the body with the use of considerably less energy than is required to bring about the chemical changes which are essential in preparing the ordinary foods for use in the human tissues. Milk is a cleaner food than meat. It is not so likely to fill the system with impurities, especially when used without the addition of other foods. Milk added to the ordinary hearty meal is of very doubtful value, though when it is apparently digested without special effort and when no unpleasant symptoms ensue, there can be no objection to this method of taking it.—*Physical Culture for May.*

## THE UNITED STATES BEHIND OTHER NATIONS IN THE SCIENCE OF INDUSTRIAL INDEMNITY.

*From Will Irwin's "The Awakening of the American Business Man" in the May Century.*

As the apostles of scientific management have shown us, we Americans have wasted foolishly in the individual processes of our industry. In the whole body of our industry we have often wasted, not only foolishly, but cruelly, and nowhere more cruelly than in the matter of provision for the wreckage of industry, the killed and wounded in our industrial warfare.

Nearly every year, perils inevitable to an age of industry kill their thousands and maim their tens of thousands. The railroads alone return an unusual list of killed and wounded employees which match well with real warfare. We have recognized dimly that either society or the industry in question owes to this wreckage some form of support for crippled, impotent years, or for a new generation of unprotected survivors.

But we are struggling along on a system of compensation for industrial accidents which is a relic of the old hand labor days, and which has worked out into a tangle of law, highly expensive, incredibly complicated, and decidedly unjust. All the so-called progressive nations entered the era of specialized labor and machine production with legal principles similar to ours; all but the United States have either amended them or changed them utterly to fit the necessities of the new age.

## SCIENCE QUESTIONS.

By Franklin T. Jones,

*University School, Cleveland, Ohio.*

*Our readers are invited to propose questions for solution—scientific or pedagogical—and to answer the questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, University School, Cleveland, Ohio.*

### Questions and Problems for Solution.

56. *Proposed by E. Carl Watson, Lafayette, Ind.*

Where is the center of pressure on the sides of a swimming tank 34 ft. long, 22 ft. wide, 4 ft. deep at one end, 7½ ft. deep at the other, filled with water? (Side is perpendicular to bottom.)

57. *Proposed by E. Carl Watson, Lafayette, Ind.*

What would be the dimensions of a rectangular resonance box to reenforce with maximum intensity—

a "C" tuning fork, rate 64 vibrations per second,
a "C" " " " 128 " " " ,
a "G" " " " 192 " " " ,
an "A" " " " 448 " " " ,
a "D" " " " 576 " " " ,
an "E" " " " 640 " " " ?

58. *Proposed by C. A. Perrigo, Dodge, Neb.*

Why does a magnet become temporarily weak upon forcibly detaching the armature?

59. *Proposed by Tom Anderson, General Chemical Co., Cleveland, O.*

How many c. c. of aqueous ammonia (sp. gr. 0.96) containing 9.90% $NH^3$ by weight, will be required to precipitate the iron as $Fe(OH)^3$ from 1 gram of $(NH^4)^2SO^4 \cdot FeSO^4 \cdot 6H^2O$?

[The following examination papers set by the International Committee of the Y. M. C. A. represent the best examinations published for *practical* men. What do you think of them?—Ed.]

### PHYSICS.

1. Write the formula for velocity. Write out in full the rules for determining velocity, space, and time derived from the formula.

2. Classify the following named forces as motor, molecular, and atomic: gravity, cohesion, affinity, adhesion, gravitation, magnetic attraction and repulsion, and electrical attraction and repulsion.

3. Give an example of a continuous force; also of an impulsive force. Which, acting alone, would result in imparting to a body uniform motion, and which accelerated motion?

4. How do you account for the great compressibility of the air? Also account for the slight compressibility of water.

5. If all moving objects on the earth's surface were to move eastward continuously, what effect would this, in time, have upon the rotation of the earth? State which one of Newton's three laws would thereby be illustrated.

6. How is the running of a pendulum clock corrected when it is gaining time, and how when losing time? What scientific principle is involved in the operation?

7. Is the invention of a perpetual motion machine a possibility? Give the scientific reason for your answer.

8. Why does a sailing vessel sink farther (or draw more water) when sailing in Lake Ontario than in the Atlantic Ocean? What is the weight of the water displaced in each case?

9. Why should the sound of one musical instrument differ from that of another, even though both may be sounding the same note?

10. In graduating the thermometer, how are the freezing and boiling points determined? What number of degrees are these points apart in the Centigrade and what number in the Fahrenheit? And where is the zero in each case?

11. Suppose a magnetic needle has been magnetized so that the marked end has south polarity; how may the correction be made so that it will have north polarity?

12. Describe and illustrate the use of the electroscope in classifying substances as conductors and non-conductors of electricity.

13. By a diagram illustrate the grouping of four dry cells in series, in parallel, and in series and parallel combined. Assuming that the voltage of each cell is 1.4 volts, what would be the voltage of the battery in each of the three groupings?

14. Name the most important part of the sounder in the electric telegraph. How is it made? If used at a long distance, which would be the better wire to use, coarse or fine? Why?

15. Show by a diagram how a person sitting in a second story room can, by using a mirror, see anyone who is at the street door.

## GENERAL CHEMISTRY.

1. What is a physical change? (1.) What is a chemical change? (2.) Tell whether a physical or a chemical change takes place when (a) salt is dissolved in water, (b) milk sours, (c) iron rusts, (d) iron is magnetized, (e) water boils, (f) mercuric oxide is heated above the boiling point of mercury. (1 credit each.)

2. If you were given five bottles, one filled with oxygen, a second with hydrogen, a third with nitrogen, a fourth with chlorine, and a fifth with carbon dioxide, how would you determine the kind of gas in each bottle? (2 each.)

3. (a) Water, blankets, and water charged with carbon dioxide are used to put out fires. Under what circumstances would each of the above mentioned be used? (2 each.) (b) Give reasons for your answer to (a). (1 each.) Why could baking soda be used to extinguish a small fire? (1.)

4. Give the meaning of (a) water of crystallization, (b) deliquescent substance, (c) efflorescent substance. (2 each.)

Mention a compound that (a) contains water of crystallization, (b) is deliquescent, (c) is efflorescent. (1 each.)

Give an illustration of a practical use of a deliquescent substance. (1.)

5. Make a drawing of the apparatus and name the chemicals that you would use for the preparation of hydrogen. (5.)

Describe an experiment that could be used to illustrate the fact that hydrogen is a reducing agent. (5.)

6. Give three proofs of the fact that air is a mixture and not a compound. (6.) What are the chief substances that air contains? (2.) Each substance you have just named constitutes what part of air? (2.)

7. Starting with an ammonium salt and a base, tell how to prepare ammonia. (5.) When ammonia is passed into water, why is some of it supposed to unite chemically with the water? (4.) What is the chemical nature of the substance formed? (1.)

8. Name a nitrate that occurs in large quantities in nature. (1.) Tell

how nitric acid is obtained from the compound named. (3.) Make a drawing of the apparatus used. (3.) Write the equation. (3.)

9. Give a test for (a) a chloride, (b) a sulphide, (c) a sulphate, (d) nitrate, (e) a carbonate. (2 credits each.)

10. What is the chief source of sulphur? (1.) Name three allotropic forms of sulphur and tell how to prepare each. (6.) Mention three important uses of sulphur. (3.)

11. Give the names of three commercial forms of carbon. (3.) Tell how each form is made. (3.) For what is each form used? (3.) What compound is formed when carbon burns in air? (1.)

12. Define the terms acid, base, and salt. (6.) Gives names of an acid and a base. (2.) Tell how to prepare a salt from the acid and base named. (1.) Give the name of the salt formed. (1.)

13. What is the chemical name of each of the following: oil of vitriol, muriatic acid, aqua fortis, blue vitriol, green vitriol, saltpeter, Chili saltpeter, Epsom salts, salt, lime. (1 each.)

14. Starting with limestone, first tell how to make lime, then tell how to slake the lime and make mortar. (6.) In a recently plastered room why would the plaster be hardened more rapidly by burning charcoal in open vessels than by heating the room by steam radiators? (4.)

15. Describe a process by which sodium hydroxide is actually made for industrial purposes. (6.) Mention two industries that consume large quantities of sodium hydroxide. (4.)

## Solutions.

45. ***Proposed by Chas. H. Korns, Bradford, Pa.***

A team of horses pulls a wagon and load up a hill. To hold the load on the hill requires a force of 4,000 pounds. Neglecting friction, do they pull more or less than 4,000 pounds, or just 4,000 pounds, in pulling it up at a uniform rate? At an accelerated rate? Do they pull any harder than the load does?

***Solution by J. G. Gwartney, Mountain View, Cal.***

(a) The team must pull the 4,000, and at first, to start the load, if started on the incline, enough more to overcome the inertia of the wagon and load. After the wagon is in motion, or if started on level ground, 4,000 and enough more to overcome the resistance offered by the air is all that is required (first law of motion).

(b) To cause the load to move at an accelerated rate, there must be a surplus force above that required to overcome the resistance offered by the air, be it ever so small.

(c) After the wagon is in motion, the team pulls no harder than the load together with the resistance offered by the air.

46. ***Proposed by O. R. Sheldon, Chicago, Ill.***

Why can one skate on "rubber ice" when the ice would break if he stood still?

***Solution by C. A. Perrigo, Dodge, Neb.***

Because the elasticity of ice is greater for a short interval than it is for long intervals.

***Solution by J. G. Gwartney.***

It takes time to transmit motion. The ice is held up by the water and cannot break until the water is driven out, but before this is done the skater is safely over.

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## WANTED.

Examination papers and questions in the sciences as a source of questions for this department. Send to the Editor.

## REAL PROBLEMS IN GEOMETRY.

EDITED BY THE REAL PROBLEM COMMITTEE OF THE C. A. S. & M. T., JAMES F. MILLIS, FRANCIS W. PARKER SCHOOL, CHICAGO, CHAIRMAN.

All of the problems below were contributed by Miss Lida C. Martin, High School, Decatur, Ill.

These problems were collected by first and second semester students of geometry in Miss Martin's classes, and for the most part are printed just as they were stated by the students. They are selected from about three hundred applied problems collected by the classes in one semester.

It is to be noted that these problems are local in their origin and application, and relate to the experiences and interests of the individual students presenting them. They represent one of the most important types of work in secondary school mathematics, supplementing the real applied problems of a more general application that may now be found in text-books, as suggested by the Real Problem Committee in its last report. Can we not have more work of the kind represented here in our secondary schools?

Many of the problems are of special interest to girls.

### Problems.

1. How can you draw a five pointed star for a sailor collar?

2. How much leather is required to cover a window seat that has three equal sides and is in the shape of a half hexagon? Length of one side is $5\frac{1}{2}$ ft., and the seat is $1\frac{1}{2}$ ft. wide.

3. Using goods 20 in. wide, how many strips will it take, cut on true bias, to put a band 12 in. wide around a skirt 3 yd. wide?

4. I have a round centerpiece with 6 in. radius. I want to stencil a design 2 in. in length, and use 6 of these figures around the circumference. How can I place them equal distances apart?

5. I want to put a new seat on a chair. The seat has a nine and a half inch radius, and I want to put 20 tacks around the circumference equal distances apart. How can I locate the positions of the tacks?

6. I have a pin cushion with a 2 inch radius. I want to sew on baby ribbon making an equilateral triangle with the vertices on the edge. How can I locate the vertices?

7. I have a round plate, and I wish to find the center of it and then paint a design on around the edge, placing it five times at equal distances apart. How can the center of the plate and the centers of the designs be located? (This can be used for six, eight, ten, and twelve figures, and on plates, centerpieces, etc.)

8. I wish to make a round doily with a scalloped edge, putting 12 scallops on it. The question is how to make the scallops equal.

9. A copper lamp shade is made of 6 trapezoids, each 3 in. at top, 6 in. at bottom, and 9 in. high. How many square feet of copper are needed?

10. To find how many skeins of floss it will take to work the edge of a centerpiece. Diameter, 20 in. There are 24 yd. in a skein, and for every 2 in. of goods it takes 1 yd. of floss.

11. How much goods will it take, 20 in. wide and cut on the true bias, to cover 3 dozen buttons 1 in. in diameter, allowing $\frac{3}{8}$ in. margin to turn under?

12. Required, to make a point at the end of a belt and get it true.

13. Required, to fold a piece of ribbon so that it will make a square corner in a yoke.

14. What is the diameter of the largest round centerpiece that can be cut from a piece of goods in the shape of a triangle each side of which is 10 in.?

15. How large a diameter must a round centerpiece have in order to stencil on it 6 designs having a diameter of 6 in. and allowing 2 in. on each side?

16. Required, to put four buttons on a waist equal distances apart.

17. ABCD is the top of a square box. To inscribe a circle on it ½ in. from sides, and then make another circle with radius 1½ in. less than the first one so that a design of a wreath of flowers may be transferred between the two.

18. A waste basket has a regular hexagon as base, and measures 54 in. around the bottom and 72 in. around the top. The slant height is 17 in. How many yards of silk 24 in. wide are required to line its interior, excluding the bottom, and allowing ½ yd. for waste?

19. How many people could you seat at a round table 54 in. in diameter when extended 4 ft., allowing 2 ft. to a person?

20. A race track ¼ mile in diameter is to be made into a ball ground. It is to be made as large as possible inside the circular track. Find how long each side can be.

21. It became necessary for a factory to increase its boiler and engine capacity 100%. A new smokestack had to be built. The old stack was circular and 15 in. in diameter. What would be the diameter of a new circular stack, provided that its capacity be correspondingly increased with the power?

22. The sides of a floor are 10 ft. and 6 ft. A man wishes to tile it with tiles in the shape of a hexagon whose sides are 6 in. How many tiles will it take?

23. What would be the length of a 1 in. round rod that would hoop a tank 12 ft. in diameter?

24. A little boy's father made him four wheels for his wagon by cutting square blocks of wood, marking their centers, and with string and pencil marking out the wheels, which he then cut with a scroll saw. How could he get the centers of the blocks?

25. A city's water main is 16 in. in diameter. How many secondary mains will it supply if the secondary mains are each 4 in. in diameter?

26. In an ideal honey comb the cells are 6 sided. Why is this? In what other regular form might they be built and yet fit snugly?

27. A baseball diamond is a square 90 ft. on a side. Find the distance across from third base to first base.

28. How would you mark off a Y. M. C. A. floor 18 yd. by 20 yd. for the 20 yd. dash? See Fig. 1.

29. To make a Y. M. C. A. emblem, ABC must be made an equilateral triangle. How construct it? See Fig. 2.

30. Having a square piece of wood which you wish to place on a lathe, how would you find the center of the end?

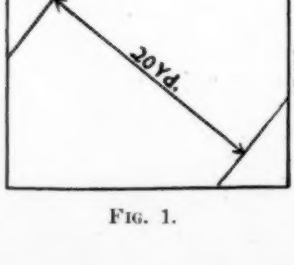


FIG. 1.

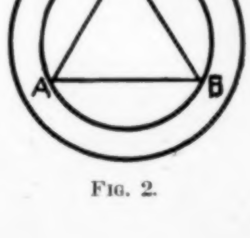


FIG. 2.

31. A circular piece of brass has a radius of 10 in. It is desired to cut a circular hole in it equal in area to one half of the disk. What should be the radius of the hole?

32. Have given one side of a vest with two pockets in it. How can the two pockets be placed parallel?

33. My father decided to buy some numerals and make the face of a grandfather's clock, but when he came to divide the face into minutes he found that he was not able to do it without guessing. How could it be done accurately?

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## ARTICLES IN CURRENT MAGAZINES.

*Nature-Study Review* for April: "Organization in Course in Nature-Study," Otis W. Caldwell; "Studies of Aquatic Insects," L. S. Hawkins; "Studying the Echo," John T. Timmons; "Photographing Water Life," Thomas I. Miller; "Farm Calendar," O. D. Center.

*American Naturalist* for April: "Genetical Studies on Oenothera," Bradley Moore Davis; "The Genotypes of Maize," George Harrison Shull.

*American Forestry* for April: "State Ownership of Forests," Austin F. Hawes; "New Ideas in Controlling Forest Fires," Samuel J. Record (with illustrations from photographs); "The Pruning of White Pine," F. B. Knapp; "Microscopic Work on the Structure of Wood," H. D. Tiemann (with illustrations from photographs); "Typical White Mountain Forest Conditions" (six illustrations from photographs of Society for the Protection of New Hampshire Forests); "Forest Fire Legislation Proposed by Wisconsin," E. M. Griffith.

*Condor* for April: "The Oasis of the Llano (with one photo), Florence Merriam Bailey; "The Blue-throated Hummingbird" (with four photos), Frank C. Willard; "Odds and Ends," Joseph Mailliard; "Doves on the Pima Reservation," M. French Gilman; "Notes on the Nesting of the Forster and Black Terns in Colorado (with seven photos), Robert B. Rockwell; "Summer Birds of Willow Creek Valley, Malheur County, Oregon," Morton E. Peck; "Nesting of the California Cuckoo in Los Angeles County, California" (with three photos), Antonin Jay; "An April Day List of Calaveras Valley Birds," Henry W. Carriger and Milton S. Ray.

*Education* for April: "A Secondary School Curriculum," B. F. Harding; "Education: The Next Phase," Charles Andrews; "Vocational Training for Girls," Isabelle McGlauflin; "The College and the Rural Districts," Wallace N. Stearns; "Waste in English Grammar," Guy Wheeler Shallies; "Moral Training of Private School Boys," Charles K. Taylor.

*Educational Psychology* for April: "The Supernormal Child," William Stern; "The New Clinical Psychology and the Psycho-Clinicist," J. E. W. Wallin; "The Spelling of College Students," William T. Foster.

*Journal of Geography* for April: "Short Studies Abroad: The Seven Hills of Rome," W. M. Davis; "Primary Aims in Geography Teaching in the Grammar Grades," Jane Perry Cook; "Geographic Influences in the Development of Illinois," Douglas C. Ridgley; "Geography in Germany and in the United States," Martha K. Genthe.

*Popular Astronomy* for May: "The Computation of the Times of Rising and Setting of the Moon," Albert S. Flint; "Further Considerations on the Origin of the Zone of Asteroids and on the Capture of Satellites," T. J. J. See; "Halley's Comet," Frederick Slocum; "The Sun as a Star," Percival Lowell; "Splashes," Arthur M. Worthington; "The Top and Gravitation," Hyland C. Kirk; "An Hypothesis of the Origin of Anti-Cyclones," W. F. Carothers.

*Zeitschrift für den Physikalischen und Chemischen Unterricht* for March: "Die physikalischen Übungen am Sophienrealgymnasium zu Berlin," P. Johannesson: "Elektro-optische Aufnahme von physikalischen Vorgängen mit dem Oszillographen," K. Fischer; "Zwei einfache, leicht selbstzufertigende Apparate zur Mechanik," J. Thiede; "Noch ein Universalgestell für Schülerübungen," K. Speyerer: "Bemerkungen zur Auswertung des Allotropiebegriffes im Unterricht," Fr. Küspert; "Über die Poggendorfsche Kompensationsmethode," K. Lichtenecker.

## PROBLEM DEPARTMENT.

By E. L. Brown.

*Principal North Side High School, Denver, Colo.*

*Readers of this magazine are invited to send solutions of the problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott St., Denver, Colo.*

### Algebra.

244. *Proposed by A. B. Carlton, Walla Walla, Wash.*

Find the value to $n$ terms of the continued fraction—

$$\cfrac{x}{1+\cfrac{x}{1+x\ldots\ldots}}$$

I. *Solution by N. Anning, Chilliwack, B. C.*

By taking a few terms at the beginning, we note that the coefficients of the powers of $x$ are the numbers in the dotted lines in Pascal's Triangle, as follows—

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| 1 | 3 | 6 | 10 | 15 | 21 | | | |
| 1 | 4 | 10 | 20 | | | | | |
| 1 | 5 | | | | | | | |

From the relation, $\dfrac{P_n}{Q_n} = \dfrac{x}{1+\dfrac{P_{n-1}}{Q_{n-1}}} = \dfrac{xQ_{n-1}}{Q_{n-1}+P_{n-1}}$

we see that each numerator must be $x$ times the preceding denominator and that any denominator $Q_m = Q_{m-1} + P_{m-1} = Q_{m-1} + x\,Q_{m-2}$.

Assume $Q_{m-1} = 1 + {}^{m-2}C_1\,x + {}^{m-3}C_2 x^2 + {}^{m-4}C_3 x^3 + \ldots\ldots$

And $\quad Q_{m-2} = \qquad 1 \quad + {}^{m-3}C_1 x + {}^{m-4}C_2 x^2 + \ldots\ldots$

Then $Q_m = Q_{m-1} + x\,Q_{m-2}$

$= 1 + ({}^{m-2}C_1 + 1)\,x + ({}^{m-3}C_2 + {}^{m-3}C_1)\,x^2 + ({}^{m-4}C_3 + {}^{m-4}C_2)\,x^3 + \ldots\ldots$

$= 1 + {}^{m-1}C_1\,x + {}^{m-2}C_2\,x^2 + {}^{m-3}C_3\,x^3 + \ldots\ldots\ldots\ldots\ldots\ldots$

This expression might have been obtained from that assumed for $Q_{m-1}$ by replacing $m-1$ by $m$. Hence if the form is right for any pair of consecutive Q's it is right for all succeeding.

Now $\dfrac{P_1}{Q_1} = \dfrac{x}{1}$, $\dfrac{P_2}{Q_2} = \dfrac{x\,(1)}{1+x}$, $\dfrac{P_3}{Q_3} = \dfrac{x\,(1+x)}{1+2x}$ &c.

$Q_2 = 1 + {}^1C_1 x$, $Q_3 = 1 + {}^2C_1 x$. These are in accord with the form chosen. Hence also are $Q_4$, $Q_5$, $Q_6 \ldots\ldots\ldots\ldots\ldots\ldots$

$$\frac{P_n}{Q_n} = \frac{x(1 + {}^{n-2}C_1 x + {}^{n-3}C_2 x^2 + \ldots\ldots)}{1 + {}^{n-1}C_1 x + {}^{n-2}C_2 x^2 + \ldots\ldots},$$

both series to run until they terminate of themselves.

II. *Solution by E. B. Escott, Ann Arbor, Mich.*

The convergents are

$$\frac{x}{1+x},\quad \frac{x+x^2}{1+2x},\quad \frac{x+2x^2}{1+3x+x^2},\ \ldots\ldots\ \ldots\ldots,\ \frac{P_n}{Q_n},\ \ldots\ldots \qquad (1)$$

where $P_n$ and $Q_n$ form recurring series with the relations

$$P_n = P_{n-1} + x\,P_{n-2} \text{ and } Q_n = Q_{n-1} + x\,Q_{n-2}\ldots\ldots \qquad (2)$$

Form the series having $P_n$ for the coefficient of $y^{n-1}$. We find the generating function to be

$$\frac{x}{1-y-xy^2} = x+xy+(x+x^2)y^2+(x+2x^2)y^3+\ldots\ldots \qquad (3)$$

But $\dfrac{x}{1-y-xy^2} = \dfrac{x}{a-b}\left(\dfrac{a}{1-ay} - \dfrac{b}{1-by}\right)$.

$\dfrac{a}{1-ay} = a+a^2y+a^3y^2+\ldots\ldots$, a geometrical progression of which the $n$th term is $a^ny^{n-1}$. Therefore the $n$th term of (3)

$$\text{is } \frac{x(a^n-b^n)}{a-b}\,y^{n-1};\text{ that is, } P_n = \frac{x(a^n-b^n)}{a-b}$$

$$Q_n = P_{n+1}/x = \frac{a^{n+1}-b^{n+1}}{a-b}.$$

Therefore the $n$th convergent to the continued fraction

$$\text{is } \frac{x\,(a^n-b^n)}{a^{n+1}-b^{n+1}},$$

where $a = \frac{1}{2}\,(1+\sqrt{1+4x})$ and $b = \frac{1}{2}(1-\sqrt{1+4x})$.

245. *Proposed by Norman Anning, Chilliwack, B. C.*

Find the general term of—

0, 0, 1, 1, 2, 4, 5, 7, 10, 12, 15, 19, ......

I. *Solution by H. E. Trefethen, Kent's Hill, Me.*

The given series may be written thus—

$0, 1, 5, 12, 22, \ldots, (3n-4)\,(n-1)/2$ (1)

$0, 2, 7, 15, 26, \ldots, (3n'-2)\,(n'-1)/2$ (2)

$1, 4, 10, 19, 31, \ldots, 3n''(n''-1)/2+1$ (3)

If $U_r$ = the required term of the given series, then $3n-2=r$, $n=(r+2)/3$; $3n'-1=r$, $n'=(r+1)/3$; $3n''=r$, $n''=r/3$. Making these substitutions in (1), (2), (3), we have—

$U_{3n-2} = (r-1)(r-2)/6$, $U_{3n'-1} = (r-1)(r-2)/6$, $U_{3n''} = r(r-3)/6+1$; and since for all values of $r$, $(r-1)(r-2)/6+2/3 = r(r-3)/6+1$, we have for the required general term

$$U_r = (r-1)(r-2)/6+(1\pm1)/3$$

the double sign being so taken as to make $U_r$ an integer.

II. *Solution by the Proposer.*

To find the general term of 0, 0, 1, 1, 2, 4, 5, 7, 10, 12, 15, 19, . . .

Find the first differences, 0, 1, 0, 1, 2, 1, 2, 3, 2, 3, 4, . . .

And the second differences, 1, -1, 1, 1, -1, 1, 1, -1, 1, 1, . . .

These latter series indicate that the terms of the first are to be taken in groups of three.

Consider the series formed by the first, fourth, seventh, . . . $(3m+1)$th terms of the above. It is 0, 1, 5, 12 . . . , a recurring series whose second differences are constant. The general term—

$$\frac{3m^2-m}{2}, \ (m=0, 1, 2\ldots\ldots),$$

is readily found and, by putting $m=\dfrac{n-1}{3}$, may be written

$$\frac{n^2-3n+2}{6}, \ (n=1, 4, 7, 10 \ \ldots\ldots).$$

Similarly the second, fifth, eighth .... terms of the given series are expressed by $\dfrac{3m^2-5m+2}{2}$, $(m=1, 2, 3\ldots\ldots)$, or by $\dfrac{n^2-3n+2}{6}$, $(n=2, 5, 8\ldots\ldots)$ where $3m-1=n$. The third, sixth, ninth .... terms of the given series are expressed by $\dfrac{n^2-3n+6}{6}$, $(n=3, 6, 9\ldots\ldots)$.

To get a single expression for the $n$th term put

$$A=\frac{n^2-3n+6}{6} \text{ and } B=\frac{n^2-3n+2}{6}$$

in the expression $\dfrac{1}{3}\left\{(A+2B)+(A-B)(\omega^n+\omega^{2n})\right\}$ where $\omega^2+\omega+1=0$. The latter is equal to A when $n$ is divisible by 3 and to B when it is not. The general term of the given series is

$$\frac{1}{18}\left\{3n^2-9n+10+4(\omega^n+\omega^{2n})\right\}.$$

## Geometry.

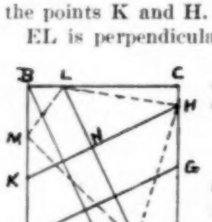
246. *Proposed by Orville Price, Denver, Colo.*

Show that any two perpendicular lines terminated by the opposite sides of a square are equal to one another, and by this property show how to escribe a square to a given quadrilateral.

*Solution by the Proposer.*

(a) ABCD is a square; KH cuts the opposite sides, BA and CD, in the points K and H.

EL is perpendicular to KH and cuts AD and BC in the points E and L.




Draw BF parallel to EL and AG parallel to KH. Clearly the triangles ABF and ADG are congruent. Therefore $AG = FB$, and $KH = EL$.

(b) Let LHEM be a given quadrilateral.

Join LE and let fall a perpendicular, HN, upon it and produce to K so that $HK = LE$. One side of the required square passes through the points K and M. Through L and E draw BC and DA perpendicular to this line through K and M, and through H draw CD perpendicular to BC. ABCD is the required escribed square.

247. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

If the incircle passes through the centroid of the triangle, find the relation between the sides $a$, $b$, $c$.

I. *Solution by the Proposer.*

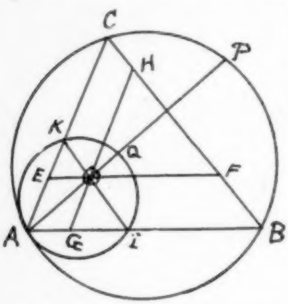
Let N be the midpoint of AC, the incircle touch AC at H and AB at P and cut BN in Q and S, $BS=2NS$. Then $BN^2=a^2/2+c^2/2-b^2/4$, . . . (1)

$BQ.BS=BP^2=(a-b+c)^2/4$ and $BS=2BN/3$.

$NQ.NS = HN^2 = (a-c)^2/4$ and $NS = BN/3$.
Hence—
$BQ.BN = 3(a^2+b^2+c^2-2ab+2ac-2bc)/8$,
$NQ.BN = 3(a^2-2ac+c^2)/4$. Adding, we have—
$BN \cdot BN = BN^2 = (9a^2+3b^2+9c^2-6ab-6ac-6bc)18 \ldots\ldots (2)$
From (1) and (2), $5(a^2+b^2+c^2) = 6(ab+ac+bc)$.

248. *Selected.*

If through a point O within a triangle ABC parallels EF, GH, IK to the sides be drawn, the sum of the rectangles of their segments is equal to the rectangle contained by the segments of any chord of the circumscribing circle passing through O.




I. *Solution by H. E. Trefethen, Kent's Hill, Me.*

EF, GH, IK are parallel respectively to AB, AC, BC. Draw chord AOP. Circle AIK cuts AP at Q. Then by problem 243—

$AO.AP = AG.AB + AE.AC$,

$AO.OP + AO^2 = AG.IB + AG.AI + AE.KC + AE.AK$,

$AO.OP = EO.OF + GO.OH + AG.AI + AE.AK - AO^2$.

Again by applying theorem 243—

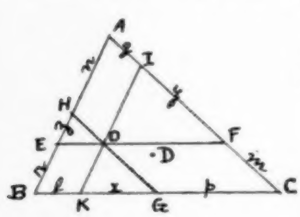
$AG.AI + AE.AK = AO.AQ = AO.OQ + AO^2$,

$AG.AI + AE.AK - AO^2 = AO.OQ = IO.OK$.

Therefore, $AO.OP = EO.OF + GO.OH + IO.OK$.

II. *Solution by N. Anning, Chilliwack, B. C.*

Name the points on the sides of the triangle in the following order around the figure—




A, H, E, B, K, G, C, F, I, A, and let D be the circumcenter.

| | | | |
|---|---|---|---|
| Put | $BK=EO=l$ | $CG=FO=p$ | $KG=x$ |
| | $CF=GO=m$ | $AI=HO=q$ | $FI=y$ |
| | $AH=IO=n$ | $BE=KO=r$ | $HE=z$ |

It is required to show that $lp+mq+nr=$ the rectangle contained by the segments of any chord of the circumcircle, drawn through O,

$=R^2-OD^2$, where R is the circumradius. From the figure, $l+x+p=a$, $m+y+q=b$, $n+z+r=c$. (1)

From similar triangles, $a:b:c = x:m:r = p:y:n = l:q:z = p+l:y+q:n+z =$ etc. (2)

From (1) and (2)

$$x = a - l - p = a - \frac{ay}{b} - \frac{az}{c}$$

$$\text{or} \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \qquad (3)$$

The point O is the centroid of masses $\frac{x}{a}$, $\frac{y}{b}$, $\frac{z}{c}$ placed at A, B and C.

For, taking moments about the line BC, if $h_a$ and $h_o$ be the distances of points A and O from this line, since $\triangle OKG$ is similar to $\triangle ABC$,

$$\frac{h_o}{h_a} = \frac{x}{a}$$

or $\frac{x}{a} h_a = h_o \times 1 = h_o \left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c}\right)$, from (3).

Therefore the centroid lies in EF and, from symmetry, must lie also in GH and IK, *i. e.* at O.

The moments of inertia about D of masses $\frac{x}{a}$, $\frac{y}{b}$, $\frac{z}{c}$ at A, B and C,

$$\frac{x}{a}AD^2+\frac{y}{b}BD^2+\frac{z}{c}CD^2=\frac{x}{a}AO^2+\frac{y}{b}BO^2+\frac{z}{c}CO^2+\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)$$
$OD^2$.

But AD=BD=CD=R and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

$$\therefore R^2-OD^2=\frac{x}{a}AO^2+\frac{y}{b}BO^2+\frac{z}{c}CO^2. \qquad (4)$$

O is the centroid of masses $p$ at E and $l$ at F because OF=$p$ and OE=$l$. Taking moments of inertia about A,

$$pAE^2+lAF^2=pl^2+lp^2+(l+p)AO^2$$
$$\text{or } (AO^2+lp)(l+p)=p(n+z)^2+l(y+q)^2$$
$$AO^2+lp=\frac{p}{l+p}(n+z)^2+\frac{l}{l+p}(y+q)^2$$
$$=\frac{n}{n+z}(n+z)^2+\frac{q}{y+q}(y+q)^2 \qquad \text{from (2)}$$
$$=n(n+z)+q(y+q)=n(c-r)+q(b-m) \qquad \text{" (1)}$$
$$=cn+bq-mq-rn$$
$$AO^2=cn+bq-(lp+mq+nr)$$

$$\text{And } \frac{x}{a}AO^2=\frac{xc}{a}n+\frac{xb}{a}q-\frac{x}{a}(lp+mq+nr)$$
$$=nr+mq-\frac{x}{a}(lp+mq+nr) \qquad \text{from (2)}$$
$$\frac{y}{b}BO^2=lp+nr-\frac{y}{b}\;(\quad\text{"}\quad) \text{ similarly}$$
$$\frac{z}{c}CO^2=mq+lp-\frac{z}{c}\;(\quad\text{"}\quad)\quad\text{"}$$

Adding,

$$\frac{x}{a}AO^2+\frac{y}{b}BO^2+\frac{z}{c}CO^2=2(lp+mq+nr)-\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)(lp+mq+nr)$$
$$=(lp+mq+nr)(2-1)$$
$$=lp+mq+nr.$$

But by (4) the first member of this equation is equal to $R^2-OD^2$

$\therefore lp+mq+nr=R^2-OD^2$=the rectangle on the segments of any chord of the circumcircle passing through O.

## Credit for Solutions Received.

Algebra 244. N. Anning, L. E. Davis, E. B. Escott, R. A. Johnson, H. E. Trefethen. (5)

Algebra 245. N. Anning, L. E. Davis, E. B. Escott, R. A. Johnson, J. Stark, H. E. Trefethen. (6)

Geometry 246. J. G. Gwartney, Orville Price, J Stark, E. R. Swartz. (4)

Geometry 247. N. Anning, L. E. Davis, R. A. Johnson, J. Stark, E. R. Swartz, H. E. Trefethen. (6)

Geometry 248. N. Anning, H. E. Trefethen. (2)

Total number of solutions, 23.

## PROBLEMS FOR SOLUTION.

### Algebra.

254. *Proposed by Laura Stiles, Denver, Colo.*

If $a$, $b$, $c$ denote positive quantities, show that

$$(a+b+c)^3>27\,abc.$$

255. *Proposed by Gustave Jacobson, Chicago, Ill.*

A corporation needing some additional capital for a short term of years, issues $300,000 of debenture bonds carrying 6% interest, and payable one fifth each year for five years. Coupons are attached to the bonds maturing every six months; the bonds are sold at 90 flat. What is the true rate of interest that the company pays for the money?

### Geometry.

256. *Proposed by A. R. Lewis, Washington, D. C.*

If two straight lines PRQ, ACB cut three concurrent straight lines OPA, ORC, OQR; then $\frac{PR}{RQ} : \frac{AC}{CB} = \frac{OP}{OA} : \frac{OQ}{OB}$.

257. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

In a quadrant, radius 10 inches, a circle is inscribed and in the space between this circle the arc of the quadrant and one of its sides a smaller circle is inscribed. From the center vertex of the quadrant a line is drawn through the center of the smaller inscribed circle to meet the tangent drawn at the middle point of the arc of the quadrant. Show that the length of the intercepted tangent is 7½ inches.

258. *Selected.*

If ABCD be a cyclic quadrilateral, and if we describe any circle passing through the points A and B, another through B and C, a third through C and D, and a fourth through D and A; these circles intersect successively in four other points, E, F, G, H, forming another cyclic quadrilateral.

## HINTS TO BEAUTY SEEKERS.

Viewed solely as a health-building measure, water bathing has wonderful restorative powers. The skin is plentifully supplied with nerves and blood vessels (remember, the skin is an organ itself, and not merely a protective covering for the tissues and organs under it), and the bath is thus of great value in stimulating the nervous system and accelerating the circulation of the blood. Water treatment in its many forms is depended upon in many great institutions in Europe exclusively in the treatment of certain diseases. The bath, in fact, is like a splendid system of exercise. It accelerates the circulation, the respiration, and the elimination of wastes from the body. It invigorates the skin, and for this very reason the entire body. The bath is really exhilarating. It is a natural stimulant and tonic.

Two other of Nature's wonderful cleansers and tonics might be mentioned—air and sun. Air is *the* important thing. No one factor causes more disease and ill-health than the lack of pure air. Pure air overcomes a multitude of hygienic shortcomings in other respects. Country people do not live more hygienically (as regards diet, bathing, exercise, and so forth) than city dwellers, but they are healthier, and the abundance of fresh, pure air which they breathe for at least nine months of the year is almost entirely responsible for this fact.—*From Physical Culture for May.*

## A BIBLIOGRAPHY OF SCIENCE TEACHING.

By W. L. Eikenberry.

A committee of the American Federation of Teachers of the Mathematical and Natural Sciences has been at work some time upon the preparation of a select list of references upon the teaching of the sciences. Their work has been issued by the Government Printing Office as Bulletin 1, 1911, U. S. Bureau of Education, under the title, Bibliography of Science Teaching.

There are some three hundred and seventy citations classified in Biology, Chemistry, Geography, Mathematics, Nature Study, and Physics. The limits as to number of references compelled very close selection and elimination. The titles that remain represent "really serious contributions to the field." The bulletin should have a large circulation. It is fortunate that it is issued in a form that makes it so easily and so generally available.

The thanks of science teachers everywhere are due to the members of this committee who have labored so effectively in this matter.

## CEMENT EXPANSION STUDIED BY CHEMICAL ENGINEERS.

Cement sidewalks grow. For that reason one must leave expansion joints at short intervals to prevent the walk from growing humpbacked. Furthermore, cement sidewalks shed their top coat often as a snake sloughs its skin. Both these phenomena are explained, says Professor A. H. White of Chemical Engineering Department of the University of Michigan, by the curious expansion of cement when moistened. The unique feature which causes the trouble is that cement once expanded by water never shrinks quite to its former dimensions upon drying, and therefore there is a constant tendency for it to increase in size. In this way a long sidewalk might grow a foot or more in ten years, though the rate of growth seems, from long time tests, to decrease with time.

The peeling of the top coat, so often noticed, is due to the fact that this coat contains almost all cement and little sand. Pure cement expands more than cement and sand, and therefore the top coat outgrows the sidewalk which it covers, and peels off.

## ANTARCTIC COAL.

The discovery of several seams of coal, a few inches to 7 ft. in thickness, in the sedimentary strata of a mountain in the Antarctic region, and of fossil remains of conifer trees by Lieutenant Shackleton's recent expedition, is a very remarkable one, and is analagous to similar discoveries of coal accompanied by fossil imprints of semi-tropical or temperate-zone foliage within the northern Arctic Circle, and in such high altitudes as those of Spitzbergen and northern Alaska, both the coal and the fossil foliage go to show that a very different climate than now, once prevailed in those regions—a climate comparable to that of Europe and the middle and southern states of North America, or even warmer. To account for such warm conditions is as difficult as to account for the present great ice-cap at the poles, and that of the glacial epoch, whether we assume a geological or an astronomical cause of these strange phenomena, such as local subsidence and elevation of land at the poles, or that the axes of the earth have changed places with regard to their position and distance from the sun and its heat.

## INDIANA ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The sixteenth annual meeting of the Indiana Association of Science and Mathematics Teachers was held in Marion on March 10 and 11.

At the first session, on Friday afternoon, Supt. J. F. Giles of the Marion schools gave the address of welcome, to which Franklin S. Lamar of the Richmond High School and president of the Association, responded.

Dr. John Hurby, secretary State Board of Health, then gave an address on "The Teacher and Public Health," in which he reviewed the very recent state legislation regarding school hygiene and sanitation.

In the evening Dr. Carl H. Eigenmann of Indiana University gave an exceedingly interesting lecture on "A Trip through British Guiana."

The business meeting was held on Saturday morning. It was decided to again leave to the decision of the executive committee the place of meeting for next year.

Officers for the coming year were elected as follows:

President, C. A. Vallance, Indianapolis; vice-president, Roy Cummins, Ft. Wayne; secretary-treasurer, Henry F. A. Meier, Evansville.

The various sections then carried out their programs, as follows:

*Physics and Chemistry Section.*

Professor J. B. Garner, Wabash College, chairman.

"Precious Stones" (illustrated), Frank B. Wade, Shortridge High School, Indianapolis; "Making and Coloring Lantern Slides," Charles S. Coons, Connersville High School; "The Girl Problem," E. A. Bureau, Muncie High School; Discussion, Principal J. M. Gray, Huntington High School; "Lectures for the Public on Chemistry," Professor W. E. Blanchard, DePauw University; "A First Year Science Course," T. A. Hanson, Kokomo High School; Discussion, E. G. Campbell, Bluffton High School.

*Mathematics Section.*

Professor H. M. Kenyon, Purdue University, chairman.

"Precious Stones," Frank B. Wade (Joint meeting with Chemistry and Physics); "The Purpose of Mathematics in the Secondary School," Byron D. Neff, Manual Training High School, Indianapolis; Discussion, Wm. Reed, Hartford City High School; "The Essential in Elementary Mathematical Education," Professor A. S. Hathaway, Rose Polytechnic Institute, Terre Haute; "The Educational Value of Mathematics," W. R. Hardman, Anderson High School; Discussion, Bessie Baer, Wabash High School; "The Introduction of Demonstrative Geometry," R. L. Modesitt, Bloomington High School; Discussion, C. E. McClintock, Lebanon High School.

*Biology and Physiography Section.*

J. F. Thompson, Richmond, chairman.

"Present Conceptions of the Protective Substances in the Blood," Dr. L. J. Rettger, State Normal School; "The New Unit in Botany," Miss Rousseau McClellan, Shortridge High School, Indianapolis; "Agriculture in the Secondary School," Professor George L. Roberts, Purdue University.

The number in attendance and the interest shown far surpassed that of any meeting held within the last five years.

HENRY F. A. MEIER, *Secretary.*

## NORTHWESTERN OHIO CENTER OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

This association was organized at Toledo in March, the first annual meeting being held in that city at the Central High School on April 29. At this meeting the following permanent officers were elected: President, C. M. Brunson; vice-president, Miss Marie Gugle; secretary-treasurer, M. R. Van Cleve—all of Toledo Central High School; executive committee, Miss Schneider, W. P. Holt, A. W. Stuart of Toledo Central High School; Miss Streimpfer of Toledo East High School; and Mr. I. F. Mattison of Delphos.

The meeting was a live one. Much enthusiasm was manifested. The matters presented and discussed were pertinent ones. Professor Fred J. Hillig of St. John's College, Toledo, read a paper on "The Ideal Text-book in Physics;" R. W. Reckard, principal of Findlay High School, presented some interesting phases of the teaching of physiography; Professor R. L. Short of Technical High School, Cleveland. gave an illuminating exposition of methods of correlating the various branches of high school mathematics; I. F. Mattison, principal of Delphos High School, spoke on the subject of teaching agriculture in the high school; Miss Isabel Wheeler of Toledo Central High School discussed "College Credit for High School Physiology."

All teachers of mathematics and science in Northwestern Ohio are invited to join this Toledo center. Communicate with the secretary.

M. R. Van Cleve.

## ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

The sixteenth meeting of the Association of Teachers of Mathematics in the Middle States and Maryland was held in Teachers College, New York, April 22, 1911. The meeting was called to order by the president, Dr. Wm. H. Metzler, at half-past ten o'clock, in the chapel of the college.

After the reading of the minutes Mr. Breckenridge, chairman of the committee on continuation schools, reported the progress of his committee. The report was very interesting in the matter of the attitude of the students in those schools for more pure mathematics merely because of their place in the curriculum of the ordinary day school. The report was accepted, and the committee was continued. The algebra syllabus committee was also continued.

The first paper of the morning was given by Mr. J. T. Rorer of the Wm. Penn H. S., Philadelphia, on "The Curriculum: Present Tendencies, Future Possibilities."

The work of the morning was concluded by a paper by Mr. A. W. Curtis of Oneonta Normal School on "Study Supervision: Its Needs in the Mathematics of the Elementary and Secondary Schools."

The first paper of the afternoon was a description with lecture table models of the slide rule and its uses, by Clifford B. Upton of Teachers College. This was followed by a description with stereopticon illustration of the calculating machines then on exhibition in the educational museum of Teachers College.

Preliminary reports for the committees on arithmetic, algebra, and geometry were given by Mr. Rorer for the committee on arithmetic and by Mr. Durrell for the committee on geometry. These reports consisted of plans for carrying on the work. After expressing its thanks to Teachers College, the meeting adjourned to the Educational Museum for the privilege of inspecting the exhibition of slide rules, calculating machines, rare books and manuscripts.

H. F. Hart, *Secretary.*

## REPORT OF THE MIDWINTER MEETING OF THE ASSOCIATION OF MATHEMATICAL TEACHERS IN NEW ENGLAND.

The midwinter meeting of the Association of Mathematical Teachers in New England was held at Worcester, Mass., in the English High School, on Saturday, March 4. The meeting was called to order at 3 P. M. by the president, Mr. Archibald V. Galbraith.

Resolutions on the death of Edgar Hamilton Nichols were read by Professor William F. Osgood, chairman of the committee. Upon motion of the chairman, the association voted to incorporate the resolutions in the minutes.

Professor Arthur Schultze, head of the Department of Mathematics in the High School of Commerce, New York, spoke upon "Causes of the Inefficiency of Mathematical Teaching." Mr. Walter B. Russell, director of the Franklin Union, Boston, spoke upon "Mathematics Required by Industrial Workers."

These two papers were followed by a general discussion upon the above topics and the general topic of the meeting, which was "The High School Course in Mathematics: What Changes are Desirable in its Present Content and Arrangement." H. D. GAYLORD, *Secretary.*

### Abstract of Professor Schultze's Address.

It is generally agreed that the present teaching of mathematics is unsatisfactory. While outsiders offer various remedies, teachers of mathematics seemed to have pinned their faith entirely upon one remedy, viz., the teaching of applied mathematics and the use of laboratory methods.

Undoubtedly such applied work will increase interest for and understanding of the work, but it seems it alone will not accomplish a reform of mathematical teaching. A too extreme adherence to these ideas may even injure the work. Elementary algebra and geometry have not a sufficient number of genuine applications. To substitute for "Henry's marbles" the height of Mt. Shasta, the number of school children in South Africa, the number of stars in Cepheus, etc., may appear interesting, but—

(1) We obtain no *genuine* applications of algebra, for nobody would find these numbers by such a method.

(2) Numerical difficulties distract the attention from the true issue.

(3) Examples of this kind as found in the best books are more likely to be done by rote than examples selected only from the algebraic viewpoint.

(4) The interest in such statistic facts fades away when they are accumulated in large numbers.

Applications from physics are very valuable, but mostly too difficult for the beginner.

In geometry the introduction of too many practical applications transforms the work into a system of calculations or mensuration. Laboratory work is very useful, but it is not mathematics and cannot replace mathematical reasoning.

The poor results of mathematical teaching are not due so such to the subject-matter taught, as to the spirit that pervades a great many of our schools. It is not a local but a constitutional disease. The true results of all subjects are just as poor as those of mathematics.

Schools are too fond of display. They are run on the spectacular plan, and only measurable results, obtained by a minimum outlay of money, are appreciated. Hence congested courses of study, the undue stress put upon examinations, crowded classes, excessive and mechanical memorizing, utter neglect of the reasoning power, etc.

In such an atmosphere mathematical instruction cannot thrive. Large numbers of utterly unprepared students have to be taught a great amount of mathematical *information* in order to pass examinations. But mathematical information has only little value. It is the mathematical reasoning that is valuable. It is the "can" and not the "know" that makes the mathematician.

We should attempt to influence these unfavorable conditions. We should try to diminish mathematical information. A good portion of the algebra and geometry that students have to know has only small value. Geometry should be a course in discovering and inventing. Students should be able to reason, to discover, not to *know* several hundred demonstrations.

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## EASTERN ASSOCIATION OF PHYSICS TEACHERS.

The fifty-eighth meeting of this wide-awake association was held Saturday, March 25, at Robinson Hall, Tufts College. The meeting was called to order by President Greenlaw, who called for the report of the secretary. This was presented by Ralph Channell, who stated that three regular meetings of the association had been held during the year, that five men from outside of the organization had delivered addresses to the association. He stated, too, that all committees had been at work and had presented interesting and valuable reports. The committee to investigate the teaching of physics in the small high school had compiled and had printed a very valuable summary of their investigation and had distributed nearly six hundred copies throughout New England. The membership remains practically the same as last year—seventy-seven active, seventy-three associate, and three honorary; total, one hundred and fifty-three. The treasury is in a healthy condition, a balance of about $75 being on hand. Reports by the chairmen of standing committees were made, the following reporting: committee on new apparatus, on magazine literature, on current events in physics, and the one on books. The following officers for the ensuing year were elected: President, C. S. Griswold, Groton School, Groton, Mass.; Vice-President, Fred H. Cowan, Girl's Latin School, Boston; Secretary, Alfred M. Butler, East Boston High School; Treasurer, Percy S. Brayton, Medford High School.

Professor Harvey N. Davis of Harvard University then spoke on, "The A B C of Aëroplane Mechanics." This paper is practically given in full on page 532 of this journal.

Professor Robert W. Willson of Harvard University then gave an interesting talk about the methods used to determine the heights of aëroplanes. At the close of this talk the meeting adjourned for luncheon. In the interim before the afternoon session many visited the museum and other college buildings.

The afternoon session was called to order by the new president, Mr. C. S. Griswold. An epidiascope was exhibited by Professor Harry G. Chase of Tufts College, who explained its action as well as showing what it was capable of demonstrating by illustrating several objects which were introduced into it.

Professor Chase also showed the Gaeda pump in operation and demonstrated the machine with tubes exhausted during the experiment. Dean Anthony invited the members to visit the shops, where several devices and pieces of apparatus were inspected.—*Abstracted from the report.*

## MATHEMATICAL AND PHYSICAL SECTION OF THE ONTARIO EDUCATIONAL ASSOCIATION.

The annual meeting of this association was held at the University of Toronto on April 18 and 19, commencing at 9:30 Tuesday morning. After registration of members and reading of the minutes, President Elliott gave an opening address, reviewing the mathematical situation in Ontario.

C. L. Crasweller of the Sarnia Coll. Inst. followed with an address on "The General Aspects of Recent Examinations." Messrs. T. A. Kirkconnell and A. M. Robertson then led a discussion on the "Entrance to Faculty of Education Papers of 1910.

Prof. W. J. Patterson of the Western University closed Tuesday's session by a paper on "The Place of Arithmetic on the High School Curriculum." The section expressed itself by resolution in favor of restoring arithmetic to the junior matriculation and entrance to normal examinations.

On Wednesday Inspector J. A. Houston gave some of his experiences in the form of a paper entitled, "Notes on Mathematics in Our Secondary Schools."

The closing address was delivered by Prof. J. L. Hogg of McMaster University, and gave the results of some research work in connection with the expansion of gases.

The following officers were chosen for the ensuing year:

Hon. President ........................J. A. Houston, Toronto
President..............................W. J. Patterson, London
Vice-President ........................R. Wightman, Toronto
Secretary-Treasurer....................A. M. Overholt, London
Councillors—W. L. Sprung, A. M. Robertson, G. W. Keith, W. W. Rutherford, and T. Kennedy

H. S. R.

## LITERARY NOTE.

Messrs. Henry Holt & Co. issued May 6 College Text-book of Physics, by Professor A. L. Kimball of Amherst College, suitable for first-year work in this subject. As far as practicable the book is non-mathematical, but it presupposes the ability to derive the fundamental formulæ. A feature of the book is a wide range of problems to test the student's grasp of the subject. The discussion of equilibrium and machines is taken up before the more difficult subject of kinetics in the belief that the historical order of development of a subject is usually the simplest mode of approach.

## NATURE'S DANGER-SIGNS.

Diseased conditions of whatever nature manifest themselves in various unmistakable external abnormal changes. These may appear in changes of contour such as swellings or depressions. Or color changes will take place—impurities always manifest their presence in muddy, turbid, yellowish or brownish shades; inflammatory states give a decided reddish hue, which deepens as the inflammation progresses. Sluggish and stagnant conditions give bluish and dark tints. Odors are symptoms that deserve notice. Offensive emanations are warnings that should be heeded. A muddy skin and offensive perspiration are relative.—*Physical Culture for May.*

## COURSES IN AGRICULTURE AT THE CORTLAND NORMAL SCHOOL.

Beginning with the fall of 1911, the State Normal and Training School of Cortland, N. Y., will offer two courses in agriculture. (1) A two-year course. (2) A one-year course.

The design of these courses is to train young men for the teaching of agriculture and the allied sciences in the public schools of the state.

*Two-year Course.*—This course is open to men at least sixteen years of age, who have had farm experience, and who have a diploma of graduation from a course (or the equivalent) prescribed by the commissioner of education for admission to normal schools (see catalog, page 13).

*One-year Course.*—This course is open to young men who are high school graduates, or have had equivalent education, have had farm experience, hold a life certificate valid in this state, and have had at least one year successful experience in teaching.

A school garden, maintained in connection with the practice school, and an eleven acre tract of splendid farm land under the control of the school offer exceptional opportunities for experimental work.

Several good dairy farms are in the vicinity of the school. The owners of these farms have assured the authorities that their herds, barns, and equipment will be at the service of the classes in agriculture.

The courses in farm crops, farm mechanics, and farm practice will include talks and demonstrations by farmers of the neighborhood.

With the rapid introduction of the study of agriculture in the public schools of the state has come a demand for teachers specially trained along these lines. Many high schools not now offering work in agriculture will do so as soon as competent teachers can be secured.

Correspondence concerning these courses is earnestly invited.

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## TIN MINED IN ALASKA.

The United States Geological Survey reports that in 1909 about 34,000 pounds of stream tin was mined in Alaska and shipped to England.

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## NATURAL GAS FIRES.

One of Stanley Waterloo's stories of prehistoric times contains a description of the region known among the cave men as the "fire country," where blue and red flames leaped from cracks in the ground. The young readers of the Story of Ab might find in "Mineral Resources of the United States for 1909," published by the United States Geological Survey, a parallel statement by United States Consul-General Michael, of Calcutta, descriptive of a real fire country, located about 20 miles from Chittagong, British India, where natural gas blazes from crevices in the ground. The gas has been burning so long that the oldest inhabitant can give no idea when or how it was set on fire. The general belief among the natives is that the gas has been on fire for centuries. At any rate, it has been burning as far back as any records have been kept by white people. It is now suggested—and some steps have been taken to carry out the suggestion—that the fire be extinguished and the gas be brought under control and piped down to Chittagong for light and fuel and power. The citizens of Chittagong have concluded that it would be cheaper to utilize the gas than to introduce electricity.

## BOOKS RECEIVED.

American Medical Association Bulletin. January issue. George H. Simmons, editor.

Standard Elocution. By I. H. Brown. 275 pages. 14x19 cm., cloth. 1911. Laird & Lee, Chicago.

Second Course in Algebra. By Herbert E. Hawkes, Columbia University, William A. Luby, and Frank C. Touton, Central High School, Kansas City, Mo. 264 pages, cloth, 13x19 cm. Price, 75 cents. Ginn & Co., Boston.

Qualitative Chemical Analysis of Inorganic Substances. By Olin F. Tower, Adelbert College. Second edition revised. 84+XIII pages. 16x24 cm., cloth. 1911. Price, $1.00, net. P. Blakiston's Son & Co., Philadelphia.

The Pupil's Arithmetic. By J. C. Byrner, Julia Richman, and J. S. Roberts. Books III and IV. Size, 7½"x5". 1911. The Macmillan Company, New York. Price, each, 35 cents, net.

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## BOOK REVIEWS.

*Graphs for Reference, by Ernest W. Ponzer, Stanford University, Cal.* Pp. 12. 11x19 cm. 1910. Price, 10 cents.

This pamphlet contains the carefully drawn graphs of 69 curves and 13 surfaces which are of frequent use in calculus, mechanics, and other related subjects. It can be used with advantage in freshman mathematics to enable the students to become familiar with some of the properties of these important curves and surfaces. H. E. C.

*Progressive Problems in General Chemistry. By Charles Baskerville and W. L. Estabrooke.* Pp. 249. D. C. Heath & Co., Boston. 1910. Price, $1.00.

This is a very fine collection of problems for college and high school students. The problems are well graded and progressive, beginning with the simple and easy and gradually increasing to the more difficult at the end of each chapter, so that the book contains material that can be used by any chemistry teacher. There are nearly three thousand problems arranged under fourteen heads. C. M. T.

*Elementary Text-book of Physics. Part I, General Physics, by R. Wallace Stewart.* Pages, 414; 187 illustrations; 14x19 cm.; cloth. 1910. $1.25, net. J. B. Lippincott Company, Philadelphia.

There is no denying the fact that this is a magnificent physics, written in a clear and readable style. A book which the layman can read with understanding and interest. It is written in very much the same style and has practically the same method of treatment as the three preceding volumes by the same author on Sound, Light, and Heat. It is free from ambiguity, there being no possible chance for wrong interpretation of the statements within its pages. A working knowledge of algebra and plain geometry is practically all the mathematics required to understand thoroughly the computations and formulæ used, except in the treatment of surface tension and capillarity is trigonometry used. It seems good to read a book and have the assurance that one can rely upon the statements made in it. This book meets the point in question. There is no better elementary treatise on the mechanics of matter in print. It is just the book for those students to use who are preparing for examination in elementary general physics. It is an English text, as well as its three companions mentioned above, which ought to be in the library of every American physics teacher. The diagrams are new, clearly executed, and to the point. Mechanically the book is well made and will stand hard usage. C. H. S.

*A Text-book of Differential Calculus.* Pp. xii+161. 14x21 cm. 1909. Price, $1.50.

*A Text-book of Integral Calculus.* Pp. x+241. 14x21 cm. 1910. Price, $1.50. *By Ganesh Prasad.* Longmans, Green & Co.

These volumes are designed to meet the requirements of Part I of the Cambridge Mathematical Tripos Examination, and the examination for the B.A. and B.Sc. degrees of Indian universities.

The author indicates some of the characteristics of the Differential Calculus as follows: "(1) The fundamental principles of the Differential Calculus have been based on a purely arithmetical foundation. Thus, the various theorems have been carefully enunciated and their proofs have been made quite independent of geometrical intuition. (2) Almost every article is followed by worked-out examples, specially suited for illustrating the article. There are also numerous exercises in every chapter. (3) A special chapter deals with curve-tracing and the important properties of the best-known curves. (4) The order in which the chapters are arranged is intended to enable the beginner to study the simple geometrical applications of the Differential Calculus immediately after he has learnt differentiation."

The chief characteristics of the Integral Calculus are: "(1) The problem of integrating a function of a single variable has been fully treated. Thus, all the types of simple functions have been considered, and the integration of each type has been discussed in a separate chapter. (2) Almost every article is followed by worked-out examples, specially suited for illustrating the article. (3) A special chapter deals with the elementary mechanical and physical applications of the Integral Calculus."

The large number of examples worked out in full, the carefully worded definitions, and the rigorous treatment throughout the text, make these volumes excellent reference books. Since the practical applications are confined to one chapter, it is probable that the author did not have in mind the needs of engineering students. H. E. C.

*The Problem of the Angle-bisectors. A dissertation submitted to the faculty of the Ogden Graduate School of Science of the University of Chicago in candidacy for the degree of Doctor of Philosophy. By Richard P. Baker.* Pp. vi+99. 22x28 cm. 1911. Price, $1.10. The University of Chicago Press.

It is probable that the problem of constructing a triangle when the lengths of the bisectors of the angles are given has been under discussion for two hundred years. Special cases have been solved, and a paper by P. Barbarin on the general case where the internal bisector formulas are used was published in 1896. But the case where the external formulas are to be used and the case where two of the assigned bisectors refer to the same vertex is not treated in general in any paper known to the author.

"The eliminations necessary in the two most difficult cases are accomplished by a combination of the method of symmetric functions and a birational transformation. A complete theory of the role of birational transformations is not at hand. In these cases one of the two parameters enters the resultant in the first order. The birational transformation reduces a rather complicated equation to one of the first order in the quantity to be eliminated. A mere substitution then accomplishes the elimination. * * *

"The separation of the roots was effected by following graphically and algebraically the plane of the sides as divided by discriminental lines through the transformations to the multiply covered plane of the parameters. * * *

"Special cases of equal bisectors, isosceles and right triangles, are discussed. In each case the group is determined." H. E. C.

*Exercises in Elementary Algebra, by May A. Blodgett.* 89 pages. 13x19 cm., cloth. 1911. Northwestern School Supply Co., Minneapolis.

Teachers of beginning algebra will find this little book very helpful in class work in furnishing a splendid list of original examples and exercises fo rhome as well as class work of their pupils. The exercises are carefully graded, there being in each topic a development from and through the simplest exercise to the more complex. The book contains many concrete illustrations of the meaning of algebraic notations. There are hundreds of exercises given, making it an easy matter for the teacher to fit the outside work to that of the class. There are numerous problems which are taken from physics and mathematics in the solution of which condensed algebraic formulas are used. The book ought to command a large and ready sale. C. H. S.

*Arithmetic, by John C. Stone, head of the Department of Mathematics, State Normal School, Montclair, N. J., and James F. Millis, head of the Department of Mathematics, Francis W. Parker School, Chicago. I. Primary Arithmetic.* Pp. xii+223. *II. Intermediate Arithmetic.* Pp. xi+240. *III. Advanced Arithmetic.* Pp. xi+249. 1911. Benj. H. Sanborn & Co.

To adapt the teaching of arithmetic to the needs and interests of the children of the present day has been the aim of the authors. All topics that do not enter some time or other into the everyday life of the average man or woman have been omitted. The problems for the upper grades are for the greater part real problems—the kind that people have to solve in everyday life—and they have been selected with the intention of keeping them within the comprehension of the pupils. In each book there is an abundance of drill exercises both oral and written.

The interest of the pupils in the primary grades is secured by an appeal to the play instinct. Games which lead to drill work in number are proposed for use in the class room, for groups of children to play outside the class room, and for one or more children to play at home. These games have been tested under the direction of the authors. Through all the grades there are many problems dealing with the various activities of the children in and out of school, and with the many phases of daily life with which children are acquainted, such as buying food and clothing, collecting mail, traveling, weighing and measuring, and so on.

The authors have based their efforts to adapt the teaching of arithmetic to the interests of children upon the following fundamental principles of education:

"I. All mental growth comes through the self-activity of the individual pupil in solving situations that to him are concrete and vital.

"II. For school work to be educative there must be genuine legitimate motive or purpose underlying it; there must be accompanying it the feeling—the positive conviction—upon the part of the pupil, that the knowledge gained is going to further his present or future interests in some way."

Book I covers the ground recommended for the second, third, and fourth grades.

Book II covers the work recommended for the fifth and sixth grades, and includes fractions, mensuration, and simple problems in percentage.

Book III covers the work recommended for the seventh and eighth grades, and includes denominate number, mensuration, proportion, percentage, powers and roots, applications of percentage, and the metric system.

These books are well printed and well bound, and with the large number of carefully drawn pictures and diagrams and the well-balanced arrangement of the reading matter, present a very attractive appearance. H. E. C.

*The Teaching of Agriculture in the High School, by G. A. Bricker.* Pp. 202. Macmillan & Co. 1911. $1.00.

The subject of agriculture is so new in the practice of our secondary schools that most teachers have rather vague ideas regarding its content, method, aims, and special pedagogy. For the benefit of the general reader, as well as for those who find themselves unexpectedly in a position which requires them to teach the subject with but slight preparation, a book is desirable which presents the pedagogical aspects of the matter. Such a book is the one now reviewed.

The author discusses the history of the subject in secondary schools, its social relations, the organization of the course, aims and methods of presentation, and educational ideals.

These chapters form stimulating reading for both sides of the controversy regarding the rightful place of agriculture, and many references to the literature tempt one to explore farther.

It is perhaps inevitable that the present very live discussion of the rights of agriculture in comparison with the older subjects should find an echo in the book. It is to be regretted, however, that the author has allowed himself to assume occasionally an argumentative attitude which detracts very much from the value of the work. The chapter on "Agriculture as a Separate Science" reads like a tract.

After all criticisms have been made, the book is yet a very welcome addition to the literature of the pedagogy of the sciences and is likely to stand for some time as a standard presentation of the pedagogy of secondary agriculture. W. L. E.

*An Introduction to Zoölogy, by Robert W. Hegner, Ph.D., Assistant Professor of Zoölogy in the University of Michigan.* Pp. xii+350. 1910. Price, $1.90. The Macmillan Company.

Dr. Hegner has presented a very good book in zoölogy, adapted as he uses it for an elementary course in college. It is valuable also as a reference book for pupils in secondary schools, although not to be chosen as a text for them. The first three chapters are devoted to the discussion of some fundamental things—the origin of life, difference between plants and animals, protoplasm, the cell and the cell theory—these subjects that are so puzzling and at the same time so fascinating to beginning students in biology. Then follow nine chapters describing types belonging to the following phyla of invertebrates: protozoa, cœlenterata, worms (flat and round), annelids, and arthropods. Echinoderms and mollusks are omitted. There is one chapter on historical zoölogy, and one on biological theories. The study of echinoderms may well be omitted from an introductory course, and reserved for a special course in invertebrate zoölogy. The study of mollusks, however, is of considerable value in a first course. Mollusks are found everywhere. They are of great economic importance. Their life histories are intimately associated with lives of other animals. The omission of mollusks leaves a bigger gap than does the omission of echinoderms. Vertebrates are omitted. The whole book is interesting. The discussion of typical animals is broad and most satisfactory. The book is not merely a morphological description, but physiological processes are well discussed in the light of the most recent research.

The author protects himself from the possible criticism of lack of originality by frankly stating in the preface, "No originality is claimed for this textbook." Except a few simple diagrams, none of the figures are his own. But in using the work of others, the author of this book has always selected the best. The book is a valuable contribution. M. B.

*The University of Chicago High School.*

*A Course of Qualitative Chemical Analysis of Inorganic Substances, with Explanatory Notes, by O. F. Tower, Adelbert College, Western Reserve University. Second edition, revised.* Pages ix+84. 1911. P. Blakiston's Son & Co. Philadelphia. Price, $1.00 net.

This is a thoroughly up-to-date laboratory manual. Fourteen pages are devoted to "a brief statement of the special principles derived from physical chemistry which are of most frequent application in qualitative analysis." The groups of the metals are named from one of the principal metals of each group instead of being numbered in the usual way. This saves much ambiguity. In acid analysis it is emphasized that the tests are tests for the acid radicals, and the symbol given is that of the corresponding ion. Explanatory notes are given on each group on the page facing the directions so that the student can most easily consult them.

Blow-pipe tests, bead tests, and flame tests are given in tabular form.

It is a manual for college students and is well adapted for advanced pupils in the high school. In many respects it is the best manual the reviewer has seen. The directions and explanatory notes are so plainly and clearly written that a minimum of explanation by the instructor is needed. C. M. T.

*Elements of Geology, by Eliot Blackwelder, Associate Professor of Geology, University of Wisconsin, and Harlan H. Barrows, Associate Professor of General Theology and Geography, University of Chicago.* American Book Company.

This book on the elements of geology is published by the same firm that published that long-used and very useful little book, a Compend of Geology, by Le Conte, the first edition of which came out in 1884. A comparison of the two books shows great advancement of the science in twenty-seven years. There are twenty-five chapters in this book, of which eight (279 pages) are devoted to physical geology and the remaining seventeen (188 pages) to historical geology, with special reference, of course, to the North American continent. The chapter headings are as follows: The Composition of the Earth, Physical Changes of the Outer Shell, The Work of the Atmosphere, The Work of the Waters Underground, The Work of Streams, Glaciers, Oceans and Lakes, The Great Relief Features of the Land, History of the Earth, Origin and Development of the Earth, The Archæozoic Era, The Proterozoic Era, and a chapter for each period from the Cambrian to the Quaternary.

Nearly five hundred halftones and line engravings and sixteen full-page topographic maps in three colors give the work an attractive appearance. The authors have written in clear language, have given numerous cross references, and have not presumed too much on the knowledge of the student. The agencies shaping the earth's surface are treated adequately and with considerable detail in the text, which is admirably supplemented by attractive illustrations germane to the subject.

In an elementary work the selection of the material to be treated in the historical part requires unusual judgment if the important mental pictures are to be given and the student's interest is to be retained. The authors have made this part about as simple as it can be made. In both parts the strong influence of the epoch-making volumes of the Chamberlin and Salisbury Geology are conspicuous.

The fullness of the treatment of the agencies and processes by which the earth's surface has been shaped makes the reader wish that the authors had gone one step further and made more use of this material in explaining, so far as may be done, the great physical features of the North American continent. A chapter on the subject at the end of the historical part, with

some emphasis on the relations of the past history to present life would not leave the student with the question, "What is all this to me?" consciously or subconsciously in mind. Such a chapter might not be appropriate for a geology designed for more advanced students, but the young student demands that he be shown that his work has some bearing on his life.

A defect in the book, but one easily remedied, is the lack of a geological map of North America. This is a serious defect, for it leaves the young student with no definite knowledge of the present surface distribution of the rocks of the different periods. When this map is supplied, a fair verdict must be that the authors have produced an admirable book.

*A Text-book of Botany for Colleges and Universities, by J. M. Coulter, C. R. Barnes, H. C. Cowles. Vol. I, Morphology and Physiology.* viii+483+L. New York, 1910. American Book Co.

Two sorts of text-books are of interest to teachers of botany in secondary schools—the high school text for use in classes and the advanced text which the teacher needs to have at hand as a reference book for himself and a source of information to the more promising of his pupils. Of high school texts the last few years have produced an abundance, and the end is not yet. With books suitable for college use and for general reference this country has not been well supplied. Several texts have appeared from American authors, it is true, but none have been generally accepted, and the premier place has probably been held by the English translation of the Bonn text-book. Science has not yet become so cosmopolitan that the use of a text written from European materials and an European viewpoint does not have very evident disadvantages.

The present text, issuing from the laboratories of the University of Chicago, is an attempt to fill the vacant place in our list of books. On the whole it is a more ambitious and comprehensive work than any preceding American text. The first volume contains the treatment of morphology from the pen of Dr. Coulter and of physiology by the late Dr. Barnes. The second volume, soon to be issued, will constitute the first general text-book of ecology which has appeared. Dr. Cowles is the author.

The morphology is written in the unusually clear and interesting style with which the other books by the same author have made us familiar. In order of presentation and in the selection of materials it develops the line marked out in Plant Structures. The same skill is used in subordinating details, while throwing into relief the principal facts and relations. No text-book can include the whole of the material now embraced in the science of botany and this one does not attempt to do so. The book may be disappointing to those who are looking for an encyclopedia of details, but it will not disappoint those who desire a modern and consistent organization of the materials of morphology for purposes of instruction or study. The teacher will find it an invaluable assistant.

The presentation of physiology exhibits the qualities of clear thinking and terse statement for which the author was notable. The arrangement of material is so carefully thought out that the index is almost superfluous. This feature will be appreciated by those who are accustomed to consult such a work as Pfeffer's. The treatment is that of the text-book rather than of the encyclopedia, but at the same time a remarkably large number of the details which one wants to know are included. A careful reading of the chapter on respiration would go far toward causing a revolution in the treatment of that subject in the schools.

The book was written for college classes, and is based upon many years' experience in classes of the grades for which it is designed. It will doubtless displace the Bonn text-book to a very great extent in college classes and in high school libraries.

The third volume will be welcomed. Certainly there is no province in botany which more needs organization than ecology, and Dr. Cowles's work should fill a "long-felt want." W. L. E.

*Standard Elocution and Speaker, by I. H. Brown.* 275 pages, 14x20 cm., extra cloth, red edges. 1911. Laird & Lee, Chicago.

"This book is a practical treatise on the science and art of vocal expression, designed for schools, colleges, universities, and private pupils. It deals with the subject in so simple a manner that no student with average zeal and ability need experience difficulty in comprehending and applying its principles. Recognizing the justice of much that has been said against the tendency of popular reciters and declamers to run into the stilted, unnatural, and affected, the author insists upon reality, intelligence, and genuine human sympathy as the basis of all delivery. The work contains specific directions for respiratory exercises, gesture, facial expressions, pronunciation, voice and speech culture, extemporaneous speaking, and rules for public speaking. A valuable feature of the work, and one that renders it of practical importance in schools, is a collection of seventy-six drills in the pronunciation of English words often mispronounced. A synopsis of the book, compactly grouping its details under general heads, aids in grasping the subject as a whole. There are numerous appropriate illustrations and a choice collection of readings and recitals in both verse and prose. This work cannot fail to fill a long-felt want, because it is accurate, practical, and modern."

C. H. S.

*Tables for the Determination of Minerals by Means of Their Physical Properties, Occurrences, and Associates, by Edward H. Kraus and Walter F. Hunt, University of Michigan.* 254 pages, 16x24 cm. $2.00 net. McGraw-Hill Book Co., New York City.

A splendid book which will be welcomed by those interested in mineralogy who desire a comparatively quick method by means of which many of the minerals may be determined. The authors have made the keynote of the work one's ability to determine "quickly the distinctions between the physical properties, occurrences, and associates of the mineral which may be suggested by the luster, color, form, streak, and hardness, especially of a given specimen." The wide experience of the writers, in both the field and laboratory, has caused them to make the color and luster of minerals the basis of the finding tables; they include 250 of the most common minerals, which are divided into three groups as very common, common, and not common, as they say the display given to the name of a mineral indicates at a glance in which one of the three groups it has been placed.

There is a brief explanation of the terms used to describe the physical properties of the specimens. A glossary of seven pages is given which contains all terms used in the introduction and in the body of the work describing the minerals. An index is found in the back containing references to the minerals only, described in the tables. In the body of the text constant reference is made to such works as Dana, Kraus and Moses, and Parsons.

This work is undoubtedly the most valuable in print for the determination of minerals by their physical properties rather than by chemical means. It is mechanically well made and arranged, and is printed in clear ten point type. It should have a large sale among those interested in geology and minerology.

C. H. S.

## SCIENCE AND MATHEMATICS SOCIETIES.

Under this heading are published in the February, June, and October issues of the journal the names and officers of such societies as furnish us this information. We ask members to keep us informed as to any change in the officiary of their society. Names are dropped when they become a year old.

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